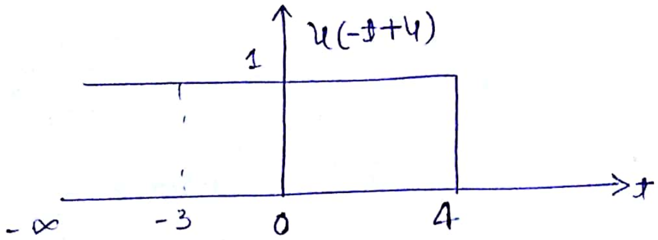
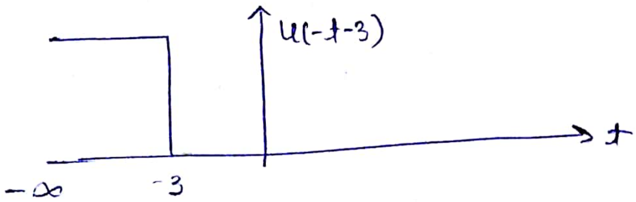
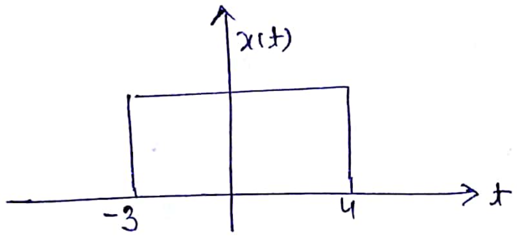
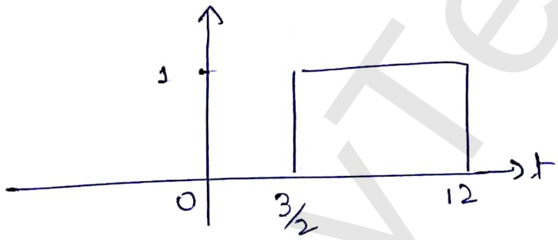
Q1 $x(t) = u(-t+4) - u(-t-3)$

$$y(t) = x\left(-\frac{2}{3}t + 5\right)$$







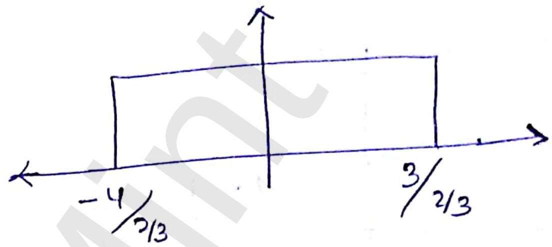


If solve from right don't take anything common.

If solve from left side take everything common.

$$y(t) = x\left[-\frac{2}{3}\left(t - \frac{5 \cdot 3}{2}\right)\right]$$

$$= x\left[-\frac{2}{3}\left(t - \frac{15}{2}\right)\right]$$

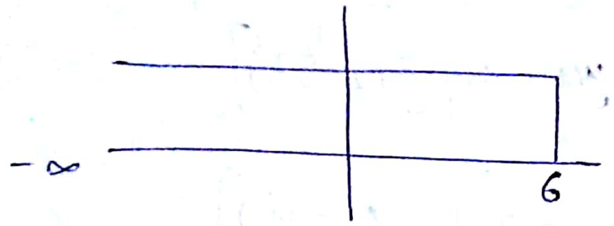


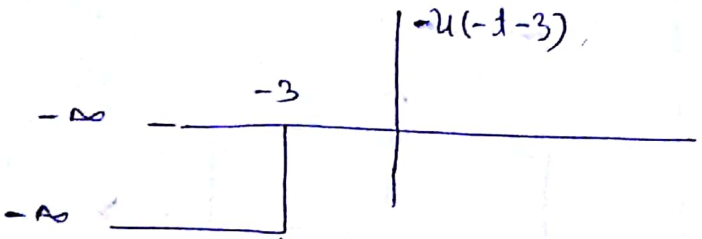
$-6 + \frac{15}{2}$ $\qquad$ $\frac{9}{2} + \frac{15}{2}$

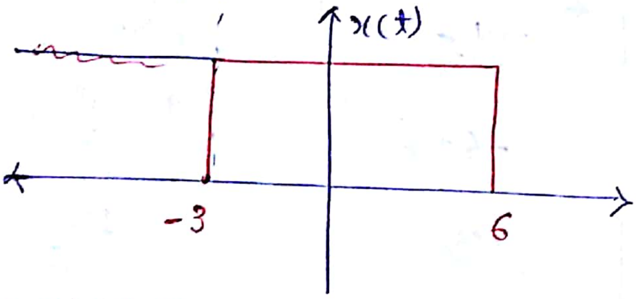
$3/2$ $\qquad$ 12

Q2 $x(t) = u(-t+6) - u(-t-3)$

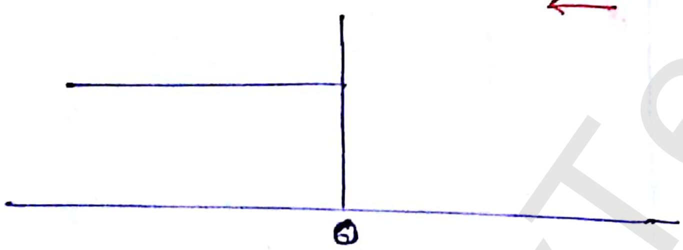
$y(t) = x\left(-\frac{4}{7}t + 6\right)$

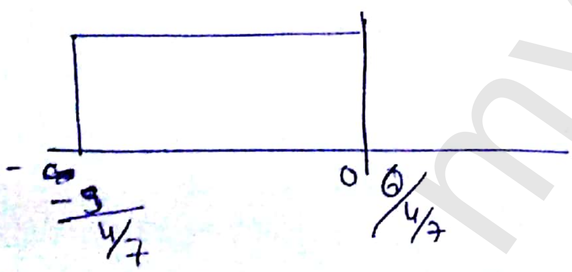


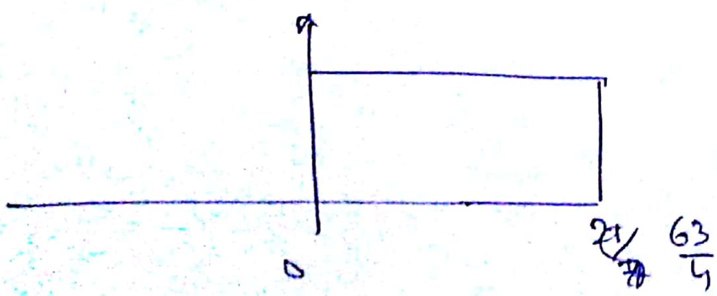




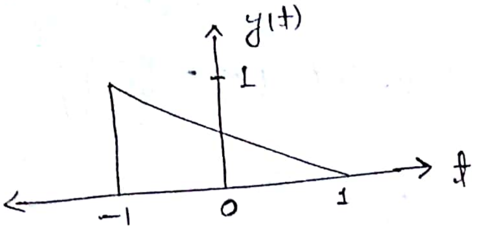
Fun right side







Q2 $y(t) = x(-2t+3) = x\left[-2\left(t-\frac{3}{2}\right)\right]$ Find x(t) ??



R to L

→ Advance by 3 → Time reversal

→ Compress by 2 → expansion by 2

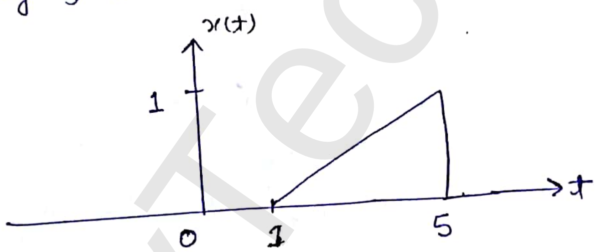
→ Time reversal → Delay by 3.

L to R

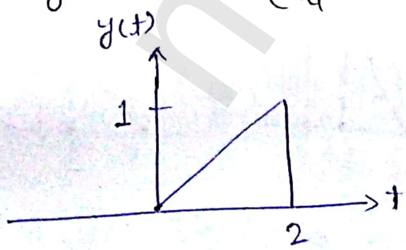
Time Reversal → Advance by $\frac{3}{2}$

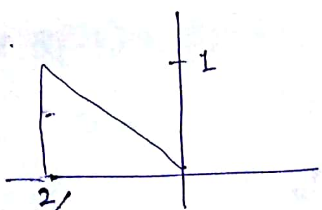
Compress by 2 → Expand by 2

Delay by $\frac{3}{2}$ → Time Reversal

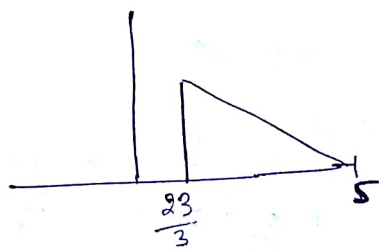


Q.23 $y(t) = x\left(-\frac{3}{4}t + 5\right)$ determine x(t)





$\frac{2}{3/4} \Rightarrow -\frac{1}{3} + \frac{5 \cdot 3}{3}$



7

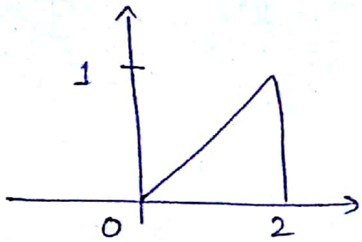
R to L

→ Advance in by 5 → T.R

→ ~~compress~~ expan by $\frac{3}{4}$ → comp by $\frac{3}{4}$

→ Time. Re. → Delay. by 5

$\frac{5}{4}+5$

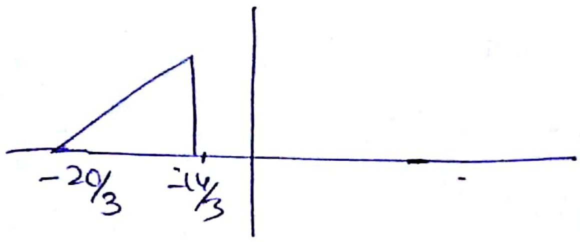


$$y(t) = x\left(-\frac{3}{4}t + 5\right)$$

$$= x\left(-\frac{3}{4}\left(t - \frac{5\cdot 4}{3}\right)\right)$$

$$=$$

L to R



TR → Adv $\frac{20}{3}$

exan $\frac{3}{4}$ → com $\frac{3}{4}$

Delay $= \frac{20}{3}$ → TR

$$\frac{3\cdot 2}{3} - \frac{20}{3}$$

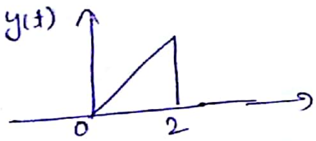
$$\frac{6-20}{3} = \frac{-14}{3}$$

$$\frac{-20\cdot 4}{3\cdot 3} \qquad \frac{-14\cdot 4}{3\cdot 3}$$

$$-\frac{80}{9} \qquad -\frac{56}{9}$$

R to L

$$y(t) = x\left(-\frac{3}{4}t + 5\right)$$

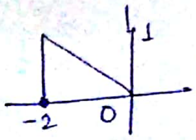


Advance by 5 → TR

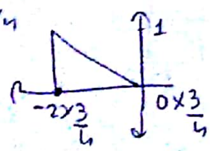
$\frac{3}{4}$ → 3/4

TR → D by 5

① TR



② 3/4



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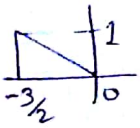
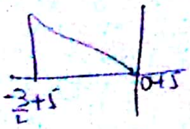


③



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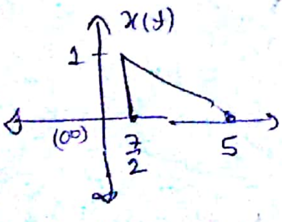
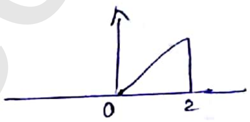


L to R

$$y(t) = x\left(-\frac{3}{4}\left(t - \frac{5\cdot 4}{3}\right)\right)$$

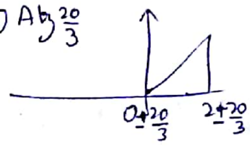


TR → A by 20/3

3/4 → 3/4

D by $\frac{20}{3}$ → TR

shift ① A by $\frac{20}{3}$



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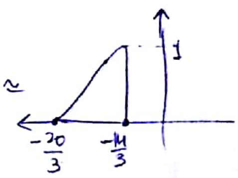
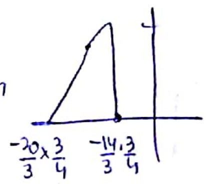


② 3/4



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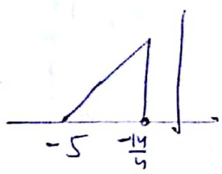
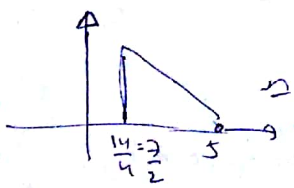


③ TR.



Q: Consider a discrete time signal $g[n]$ given by

$$g[n] = \{6, 4, 2, 0, -2, -4, -2, 0, \underset{\uparrow}{2}, 2, 2, 2, -2, -2, -2, -2, 2\}$$

if $h[n] = g[2n-3]$

$x[n] = h[n] - h[n-1]$

$$y[n] = \sum_{m=-\infty}^{n} h[m]$$

① determine the numerical values of $h[-2] =$

$$g[n] = \{6 \; 4 \; 2 \; 0 \; -2 \; \underset{\uparrow}{-4} \; -2 \; 0 \; \underset{\uparrow}{2} \; 2 \; 2 \; 2 \; -2 \; -2 \; -2 \; -2 \; 2\}$$

(i) $h[-2] = 4$

ii) $x[-1] = h[-1] - h[-2] = 0 - 4 = -4$

iii) $y[2] = \sum_{m=-\infty}^{2} h[m] = 4 + 0 - 4 + 0 + 2 = 2$

Q $g[n] = \{2, 6, -4, 0, 4, -6, -4, 0, \underset{\uparrow}{2}, 4, 6, 6, 2, 2, -4, 4, 6\}$

$\uparrow$

~~g h[n]~~

i) $h[-2] = g[2\cdot(-2)-3] = g[-7] = 6$

ii) $x[n] = h[-1] - h[-2] = g[-5] - g[-7]$

$= 0 - 6 = -6$

iii) $y[2] = \sum_{m=-\infty}^{2} h$

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Formulae

* $\delta(\alpha t) = \frac{1}{|\alpha|} \cdot \delta(t)$

* $\delta[\alpha(t \pm \beta)] = \frac{1}{|\alpha|} \delta(t \pm \beta)$

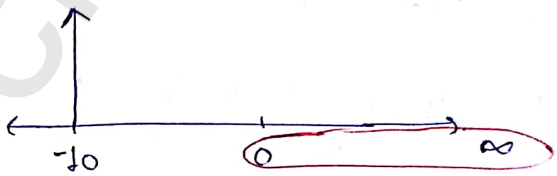
* $x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$
* $x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$

* $x(t) \cdot \delta(t + t_0) = x(-t_0)\, \delta(t + t_0)$

* $\int_{-\infty}^{+\infty} x(t) \cdot \delta(t)\, dt = x(0)$

* $\int_{-\infty}^{+\infty} x(t) \cdot \delta(t - t_0)\, dt = x(t_0)$

Q5 Let $y(t) = \int_{0}^{\infty} e^{-\alpha t^2}\, \delta(t + 10)\, dt$



$= 0$

Q.6 $y(t) = \int_{-\infty}^{+\infty} e^{-(t-1)} \cdot \sin\left(\frac{\pi}{4}(t+5)\right) \cdot \delta(1-t)\, dt$

$e^{-(0)} \cdot \sin \frac{\pi}{4} \cdot 6 \int_{-\infty}^{+\infty} \delta(1-t)$

$= \sin \frac{3\pi}{2} \cdot 1$

$= -1$

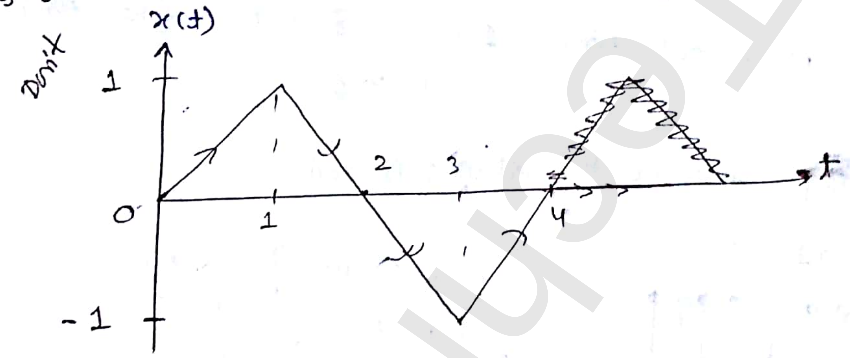
Q7 $y(t) = \int_{-\infty}^{+\infty} \left[u(t-6) - u(t-10) \right] . \sin \frac{3\pi}{4} t . \; \delta(t-5) \, dt$

$\left\{ u[-1] - u[-5] \right) . \; . \; . \; .$

$\int_{-\infty}^{+\infty} 0 . \quad \delta(t-5)$

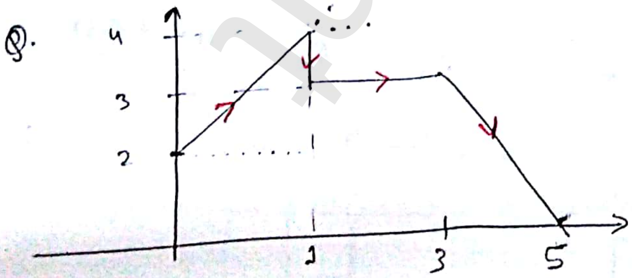
$= 0$

Q. 8 $x(t)$ shown below write expression of $x(t)$ in terms of $u(t), r(t), \delta(t)$



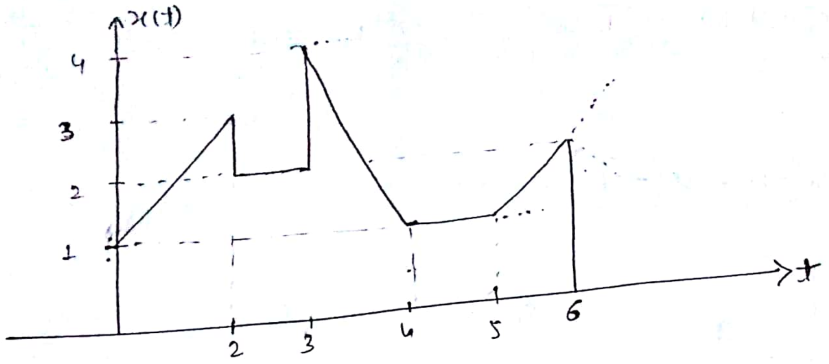
$x(t) = \quad 1.r(t) - 2r(t-1) + 2r(t-3) - r(t-4)$

Q.



$x(t) = 2u(t) + \frac{2}{1}r(t) - 2r(t-1) - u(t-1) - \frac{3}{2} r(t-3) + \frac{3}{2} r(t-5)$

Q9 a
by unit.



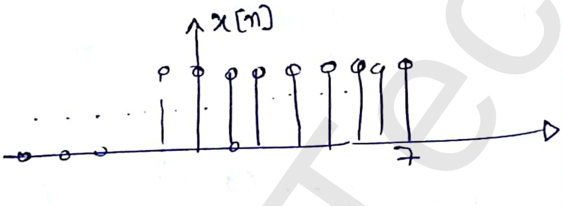
$$x(t) = u(t) + r(t) - r(t-2) - u(t-2) + 2u(t-3) - \frac{3}{1} r(t-3) + 3r(t-4)$$
$$+ r(t-5) - r(t-6) - 2u(t-6)$$

Q10 consider a descrete time signal $x(n) = 1 - \sum_{k=8}^{\infty} \delta(n-k)$ also

DO→

$x(n) = u[Mn - n_0]$ determine the values of M and n_0.

Soln



write directly
$x(n)$ is $-\infty$ to 7
$x(n) = u[-n+7]$

solve hereafter

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$u[-n+7] = \begin{cases} 1 & n \leq 7 \\ 0 & \end{cases}$$

$n - 7 \leq 0$ $\quad -n+7 \geq 0$

$-n+7 < 0$

$u[Mn - n_0] = u[-n+7]$

$M = -1$, $n_0 = -7$

don't do this

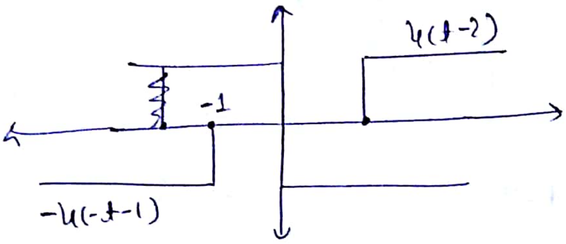
Q11. Consider a continuous time sig $x(t)$ given by

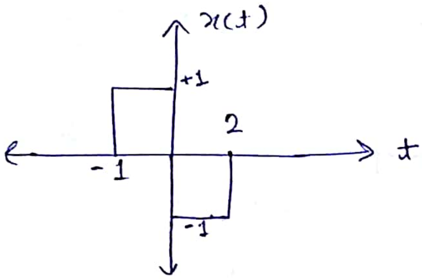
$$x(t) = -\operatorname{sgn}(t) - u(-t-1) + u(t-2)$$

Sketch

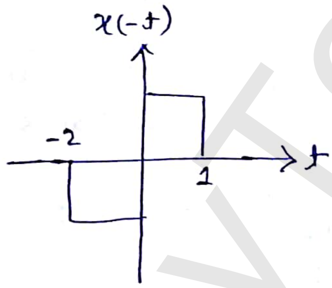
(1) $x(t)$

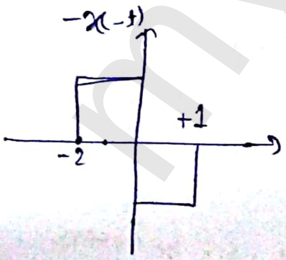
(2) $x_o(t)$

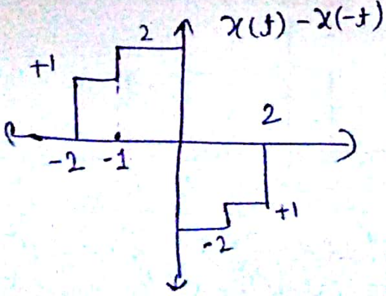


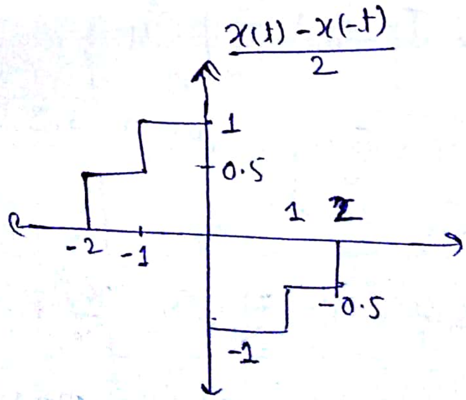


$$x_o(t) = \frac{x(t) - x(-t)}{2}$$









Q[12] Let $x(t) = 3\sin\left[\frac{2\pi}{5}(t-T)\right]$. determine the values of T for which
Do?
the resulting s/g will be :
① an even fun of time (t).
② an odd fun of time (t).

for even $x(t)$

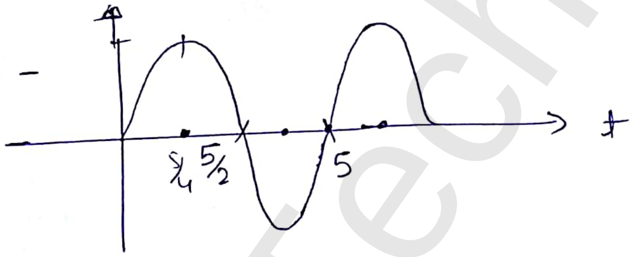
$$x(t) = x(-t)$$

$$3\sin\left[\frac{2\pi}{5}(t-T)\right] = \sin\left[\frac{2\pi}{5}(-t-T)\right]$$

$t-T$

$$\frac{2\pi}{5}t - \frac{2\pi}{5}T = -\frac{2\pi}{5}t - \frac{2\pi}{5}T$$

$\frac{2\pi}{5}t$ -



$+\cos\omega_0 t$] Both are even
$-\cos\omega_0 t$]

$\cancel{\frac{2.5}{2} = t}$ I need to change

$\cancel{\omega = \frac{2\pi}{T}}$

$\sin \cancel{\omega_0 t} \left(\frac{2\pi}{5}\right.$

for $T = \frac{5}{4}, \frac{15}{4}, \frac{+25}{4}$

$\boxed{T = \frac{5n}{4}}$ for $n = 1, 3, 5$ S/g will be even

For odd fun. $T = \frac{5}{2}, 5, \frac{3 \cdot 5}{2}, \ldots$

$= n\left(\frac{5}{2}\right)$ $n = 1, 2, 3 \ldots$

By Sir

for even

$$x(t) = 3 \sin\left[\frac{2\pi}{5} t - \frac{2\pi}{5} T\right]$$

$$\frac{2\pi}{5} T = \pm(2n+1)\frac{\pi}{2} \; ; \quad n = 1, 2, 3, 4$$

$$T = \pm(2n+1)\frac{5}{4}$$

$$T = \pm\frac{5}{4}, \pm\frac{15}{4} \pm \frac{25}{4} \pm \frac{35}{4}$$

for odd.

$$\frac{2\pi}{5} T = + n\pi \; ; \quad n = 0, 1, 2, 3 \ldots$$

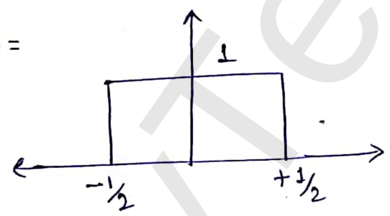
$$T = \pm \frac{5n}{2}, \quad n = 0, 1, 2, 3$$

$$T = 0, \pm\frac{5}{2}, \pm 5, \pm\frac{15}{2}, \ldots$$

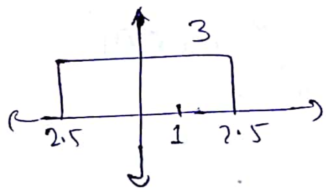
Q13 Don't Do.

$$x(t) = -4\,\mathrm{rect}\left(\frac{t+1}{4}\right) + 3\,\mathrm{rect}\left(\frac{t}{5}\right)$$ - determine. energy of sig.

rect(t) =



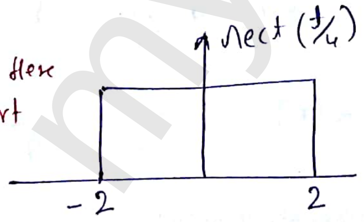
$3\,\mathrm{rect}\left(\frac{t}{5}\right)$

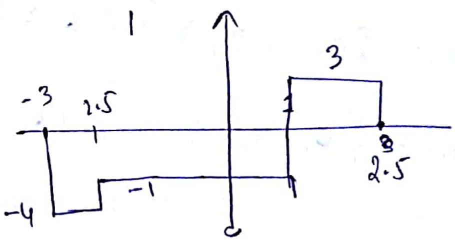


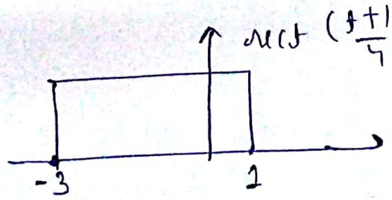
$\mathrm{rect}\left(\frac{t+1}{4}\right)$

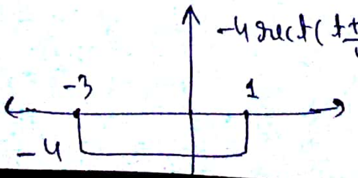
$\frac{1}{4}$ is taken common here so we need to start from left

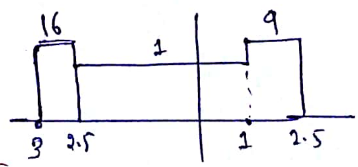
if we want to start from Right then $\mathrm{rect}\left(\frac{t}{4}+\frac{1}{4}\right)$











$Ex =$

$$= \cancel{16}\,8 \times \frac{1}{2} + 3.5 + \cancel{...}\,\frac{3}{2} \times 9$$

$$= 8 + 3.5 + \frac{27}{2} = 11.5 + 13.5 = 25\,J$$

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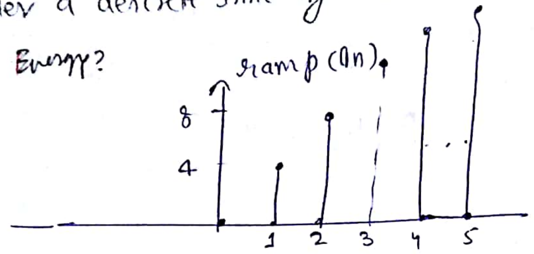
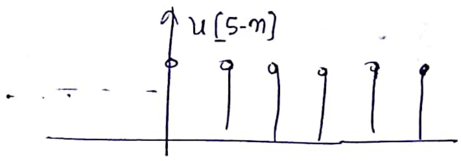
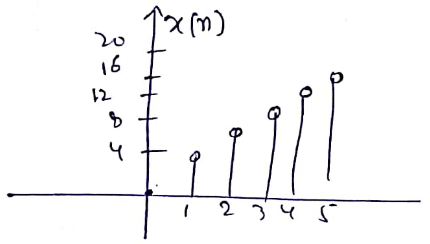
Q14 consider a descrete time sig $x[n] = ramp[4n] \cdot u[5-n]$

calc Energy?







$$E_x = 16 + 64 + 144 + \cancel{196}^{256} + 400$$

$$\cancel{830} = 880 \text{ Joule.}$$

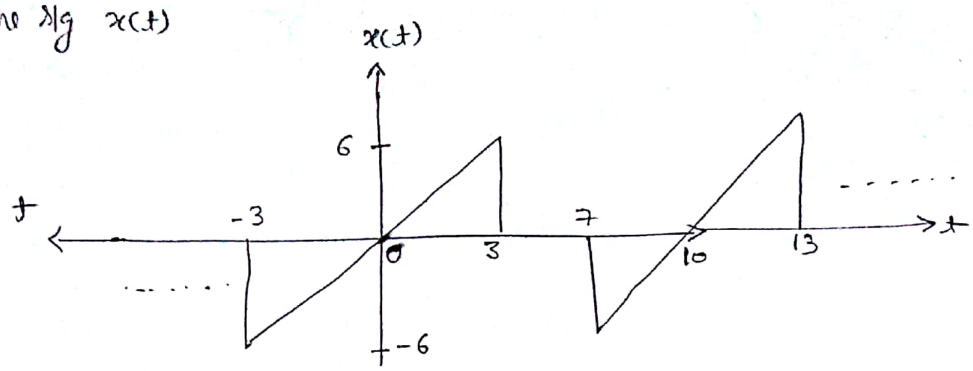
$$E_x = \sum_{n=-\infty}^{+\infty} |x(n)|^2$$

~~r[n] = {0, 1~~

$$r[n] = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, \ldots\}$$

$$r[4n] = \{0, 4, 8, 12, 16, 20\} = x(n)$$

Q.15 For the s/g $x(t)$



① determine Fundamental period of the s/g.

② " freq in Hz.

③ Power of the s/g

④ rms value of the s/g.

soln

$T = 10$ sec

$f = \frac{1}{10}$ Hz

Power = $\frac{1}{10}\int_0^3 2\cdot(2t)^2\,dt$ ~~$P_x = \frac{1}{T}$~~

$= \frac{2\cdot 4}{10}\int_0^3 t^2\,dt$

$= \frac{8}{10}\left[\frac{t^3}{3}\right]_0^3$

$= \frac{8}{10}\cdot\frac{27}{3} = 7.2$

rms = $\sqrt{P} = \sqrt{7.2}$

Chap^r-2

Q.1 $\quad y(t) = x^2(t)$

$$x_1(t) \xrightarrow{T} y_1(t) = x_1^2(t)$$

$$x_2(t) \xrightarrow{T} y_2(t) = x_2^2(t)$$

$$y_1(t) + y_2(t) = T\{x_1(t) + x_2(t)\}$$

$$= [x_1(t) + x_2(t)]^2$$

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = T\{x(t)\}$$

Q.2 $\quad y(n) = a\,x(n) + b$

Q.3 $\quad y(t) = \cos[x(t)]$

Q.4 $\quad y(t) = \cos \omega_0 t \cdot x(t)$

Q.5 $\quad y[n] = n \cdot x[n]$

2 $\quad x_1[n] \longrightarrow y_1[n] = a x_1[n] + b$

$$x_2[n] \longrightarrow y_2[n] = a x_2[n] + b$$

$$\{x_1[n] + x_2[n]\} \longrightarrow (y_1[n] + y_2[n]) \neq a[x_1[n] + x_2[n]] + b$$

3. $\quad y(t) \quad x_1(t) \longrightarrow \cos(x_1(t))$

$$x_2(t) \longrightarrow \cos(x_2(t))$$

$$x_1(t) + x_2(t) \longrightarrow \cos(x_1(t)) + \cos(x_2(t)) \neq \cos[x_1(t) + x_2(t)]$$

4. $\quad x_1(t) \longrightarrow \cos \omega_0 t\, x_1(t)$

$$x_2(t) \longrightarrow \cos \omega_0 t\, x_2(t)$$

$$x_1(t) + x_2(t) \longrightarrow \cos \omega_0 t\, x_1(t) + \cos \omega_0 t\, x_2(t) = \cos \omega_0 t\,(x_1(t) + x_2(t))$$

$y\ x_1(n) = \quad n\,x(n)$ — Linear

Q. $y(t) = Re\{x(t)\}$ Tell about L or NL

$x_1(t) \xrightarrow{T} y_1(t) = Re\{x_1(t)\}$

$x_2(t) \xrightarrow{T} y_2(t) = Re\{x_2(t)\}$

$y_1(t) + y_2(t) \longrightarrow = T\{x_1(t) + x_2(t)\}$

$= Re\{x_1(t) + x_2(t)\} = Re\{x_1(t)\} + Re\{x_2(t)\}$

Let $a = 3+j4$

$(3+j4) \cdot Re\{x(t)\} = Re\{(3+j4)\cdot x(t)\}$

Q3 $y(t) = x(-t)$ Tell about TV or TIV

$y(t-t_0) = x(-t-t_0)$

$= x[-(t-t_0)]$

$x(t) \xrightarrow{t_0} x(t-t_0) \longrightarrow x(-t-t_0)$

$x(-t-t_0) \neq x[-(t-t_0)] \Rightarrow TV$

$y(t) = x(2t)$

$y(t) = x(t/2)$

$y(t) = x(t^2)$ — TV.

$y(t) = t \cdot x(t)$

$y(t) = \frac{t}{x(t)}$

$$\boxed{Q.1 \quad y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau}$$

Find TV or TIV ?

$$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau \quad \text{—(i)}$$

$$x(t_0) \xrightarrow{t_0} x(t-t_0) \xrightarrow{T} \int_{-\infty}^{t} x(\tau - t_0)\, d\tau \quad \text{—(ii)}$$

put $\tau - t_0 = m$

$d\tau = dm$

$\tau = -\infty \qquad m = -\infty$

$\tau = t \qquad m = t - t_0$

$$= \int_{-\infty}^{t-t_0} x(m)\, dm \quad \text{—(iii)}$$

(i) = (iii) $\Rightarrow$ TIV

$$\boxed{Q.2 \quad y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau}$$

Find Time Variant or not.

$$y(t-t_0) = \int_{-\infty}^{2(t-t_0)} x(\tau) d\tau \quad \text{—(i)}$$

$$x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{T} \int_{-\infty}^{2t} x(\tau - t_0)\, d\tau \quad \text{—(ii)}$$

$\tau - t_0 = m$

$d\tau = dm$

$\tau = -\infty \qquad m = -\infty$

$\tau = 2t \qquad m = 2t - t_0$

$$\int_{-\infty}^{2t-t_0} x(m)\, dm \quad \text{—(iii)}$$

Bcz i $\neq$ iii so TV

Q Consider a descrete time system with i/p o/p eq^n given by

$y(n) = 2^{-n} \sum_{k=-\infty}^{n} 2^{k} \cdot x[k]$ determine the system LTI or not

(2) if LTI what is the impulse response of the system.

Sol^n =

$x_1[n] \longrightarrow y_1[n] = 2^{-n} \sum_{k=-\infty}^{n} 2^{k} x_1[k]$

$x_2(n) \longrightarrow y_2[n] = 2^{-n} \sum_{k=-\infty}^{n} 2^{k} x_2[k]$

$x_1(n) + x_2(n) \longrightarrow y_1(n) + y_2(n) = 2^{-n} \sum_{-\infty}^{n} 2^{k} [x_1(k) + x_2(k)]$

$a x_1(n) \longrightarrow y_1(n) = a 2^{-n} \cdot \sum_{k=-\infty}^{n} 2^{k} x_1[k]$

Linear

$y[n-n_0] = 2^{-(n-n_0)} \sum_{k=-\infty}^{(n-n_0)} 2^{k} x[k]$. (i)

$x(n) \xrightarrow{n_0} x[n-n_0] \longrightarrow 2^{-n} \sum_{k=-\infty}^{n} 2^{k} x[k-n_0]$ —(ii)

~~k-k_0 = m~~ $k - n_0 = m$

$k = -\infty$; $m = -\infty$

$k = n$; $m = n - n_0$

$= 2^{-n} \sum_{-\infty}^{n-n_0} x[m] \cdot 2^{m} \cdot 2^{n_0}$

$= 2^{-n} \cdot 2^{n_0} \sum_{-\infty}^{n-n_0} 2^{m} x[m]$

$= 2^{-(n-n_0)} \cdot \sum_{m=-\infty}^{n-n_0} 2^{m} \cdot x[m]$ —(iii)

(i) = (iii) ⇒ TIV

(2) $$y(n) = 2^{-n} \sum_{k=-\infty}^{n} 2^{k} \cdot x[k]$$

$x(n) \longrightarrow \delta(n)$

$y(n) \longrightarrow h(n)$

$$h(n) = 2^{-n} \sum_{k=-\infty}^{n} 2^{k} \cdot \delta[k]$$

$$= 2^{-n} \cdot \sum_{k=-\infty}^{n} \delta[k]$$

$$= \left(\frac{1}{2}\right)^{n} u(n)$$

Doubt. What happened with 2^k.

Most impr property ****

$$\boxed{\begin{aligned} u(t) &= \int_{-\infty}^{t} \delta(\tau)\,d\tau \\ u(n) &= \sum_{k=-\infty}^{n} \delta(k) \end{aligned}}$$

Gate. Q. $y[n] = x[n] + (-1)^{n} x[n]$

Tell this is invertible or not.

Let $x_1(n) = \{1\ 1\ 1\ 1\}$

$y_1(n) = \{1, 1, 1, 1\} + \{1, -1, 1, -1\} = \{2\ 0\ 2\ 0\}$

Let $x_2(n) = \{1, -1, 1\ -1\} = \cancel{\{2\ 0\ 2\ 0\}}$

$y_2(n) = \{1, -1, 1, -1\} + \{1\ 1\ 1\ 1\}$

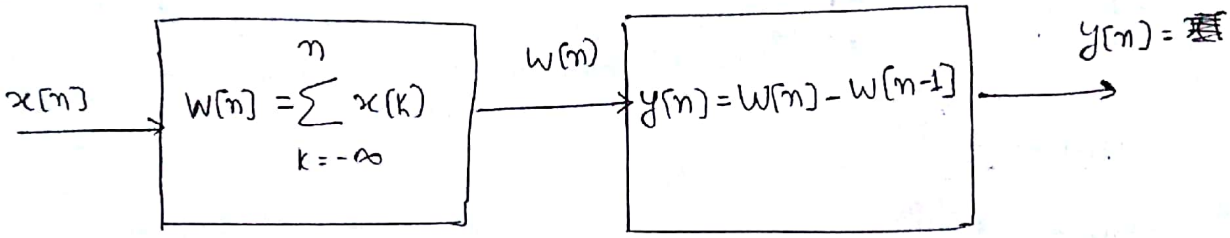
$= \{2\ 0\ 2\ 0\}$

Here for the two i/p $x_1(n) = \{1\ 1\ 1\ 1\}$ and $x_2[n] = \{1\ -1\ 1\ -1\}$ the o/p is same i.e $\{2\ 0\ 2\ 0\}$

i/p o/p

$x_1, x_2 \longrightarrow$ same ⇒ Non-invertible system

Q. Consider the cascading of two systems as shown below



determine weather system are Invertible or not

if invertible then $y(n) = x[n]$

method ① $y[n] = W(n) - W(n-1]$

let $x[n] = \delta[n]$

$$W(n) = \sum_{-\infty}^{n} \delta[n] = u[n]$$

$$W(n-1) = u[n-1]$$

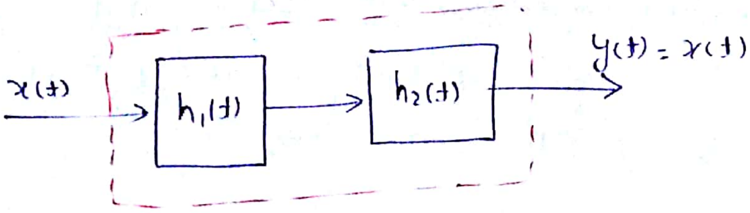
$$y(n) = W(n) - W(n-1)$$
$$= u(n) - u(n-1)$$
$$y(n) = \delta[n]$$

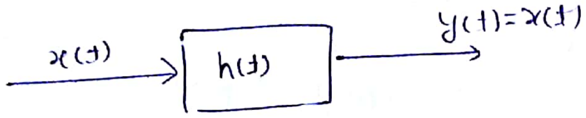
i/p $x(n) = \delta(n)$ and $y(n) = \delta(n)$ also.

method-② $W(n) = \sum_{k=-\infty}^{n} x[k]$

$$y(n) = W(n) - W(n-1)$$
$$= \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k]$$

$\boxed{y(n) = x(n)}$ ⇒ Invertible.

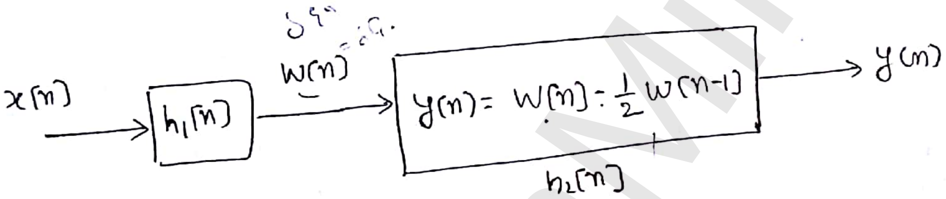




$$h(t) = h_1(t) * h_2(t) = \delta(t)$$

$$h[n] = h_1[n] * h_2[n] = \delta[n]$$

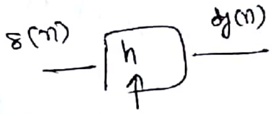
Q. Consider the cascading of two systems as shown below



$$h_1[n] = \left(\tfrac{1}{2}\right)^n u[n]$$

① determine overall impulse response of the system and ~~deter~~

② " " o/p $y[n]$ in terms of overall i/p $x[n]$.



Soln

$$h_1(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

$$h_2[n] = \delta[n] - \tfrac{1}{2}\delta[n-1]$$

$$h(n) = h_1(n) * h_2(n) = \left(\tfrac{1}{2}\right)^n u(n) * \delta(n) - \left(\tfrac{1}{2}\right)^n \cdot \left(\tfrac{1}{2}\right) u(n) * \delta[n-1]$$

$$= \left(\tfrac{1}{2}\right)^n u[n] - \left(\tfrac{1}{2}\right)^{n+1-1} u[n-1]$$

$$= \left(\tfrac{1}{2}\right)^n u(n) - \left(\tfrac{1}{2}\right)^n u[n-1]$$

$$= \left(\tfrac{1}{2}\right)^n \left[u(n) - u[n-1]\right]$$

$$= \left(\tfrac{1}{2}\right)^n \delta[n]$$

$$= \delta[n]$$

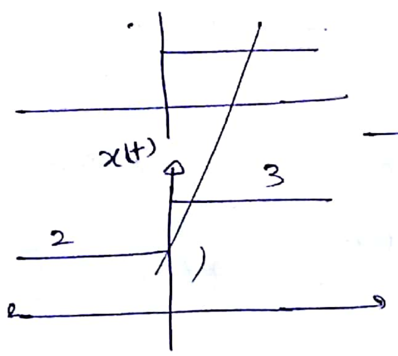
$$\left(\tfrac{1}{2}\right)^n u(n) * \delta[n-1]$$

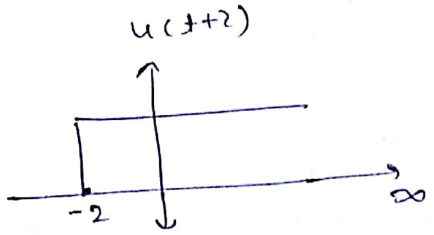
$$\left(\tfrac{1}{2}\right)^{n-1} u(n-1)$$

② $x[n] = y[n]$

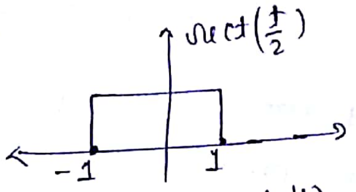
Q In a relaxed LTI system, the i/p is given by $x(t) = \cancel{u(t) + 2}\ u(t+2)$ and the impulse response is given by $h(t) = rect\left(\frac{t-4}{2}\right) + rect\left(\frac{t+4}{2}\right)$.
For what range of time t is the o/p $Y(t)$ is non zero.

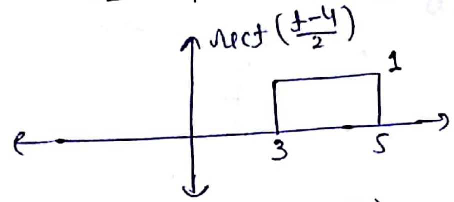
Sol^n

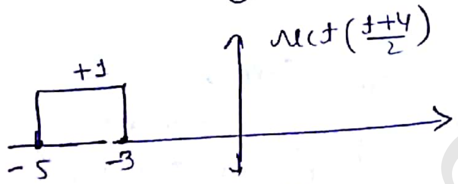


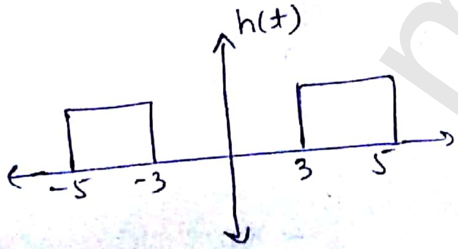


~~rect((t-4)/2)~~









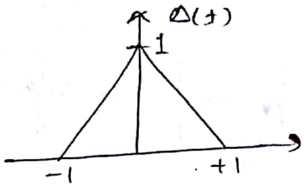
~~exis~~

$Y(t)$ exist b/w $-5+(-2)$ to ∞

-7 to ∞

Q. In a LTI system, the i/p $x(t) = rect\left(\frac{t-1}{4}\right)$ and $h(t) = 4\,\Delta(t)$

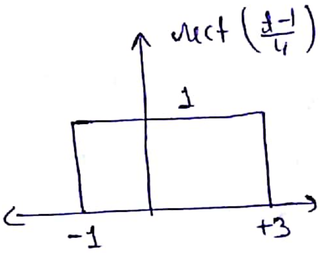
$\Delta(t) =$



What is the value of $\int_{-\infty}^{+\infty} y(t)\,dt$ where $y(t)$ is the o/p of the system.

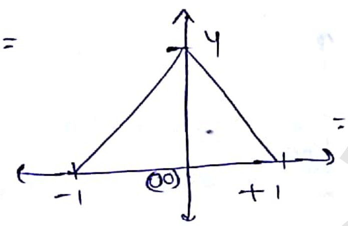
$\Delta(t) = A\,tri\left(\frac{t}{\square}\right)$ ← width

$x(t) =$



$Area_1 = 4$

$\Delta(t) =$

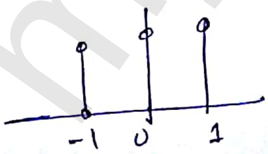


$= \quad Ar_2 = \frac{1}{2} \times 2 \times 4 =$

$$\int_{-\infty}^{+\infty} y(t)\,dt = Ar_1 \times Ar_2 = 4 \times 4 = 16$$

Q. Consider a descrete time LTI system with $h[n] = \left(\frac{1}{2}\right)^n u[n]$. if i/p $x[n] =$ $x(n) = u(n+1] - u[n-2]$ then the value of $y[5] = ?$

$x[n] =$



$= \delta(n+1) + \delta[n] + \delta[n-1]$

$y(n) = x(n) * h[n]$

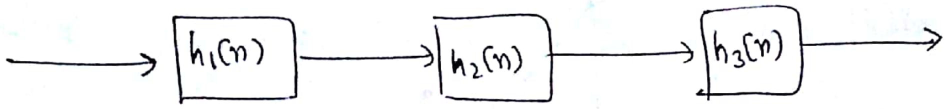
$y[5] = \left(\delta(n+1) * \left(\frac{1}{2}\right)^n u(n)\right) + \left(\delta(n) * \left(\frac{1}{2}\right)^n u(n)\right) + \left[\delta(n-1) * \left(\frac{1}{2}\right)^n u(n)\right]$

$= \left(\frac{1}{2}\right)^{n+1} u(n+1) + \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u[n-1]$

$y(5) = \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^4 = \frac{1}{2^6} + \frac{2}{2^6} + \frac{4}{2^6} = \frac{7}{2^6} = \frac{7}{64}$

Q Consider the cascading of three systems shown below

Sol-



$h_1(n) = \left(\frac{1}{2}\right)^n u(n)$

$h_2(n) = u(n+3)$

$h_3(n) = \delta(n) - \delta(n-1)$

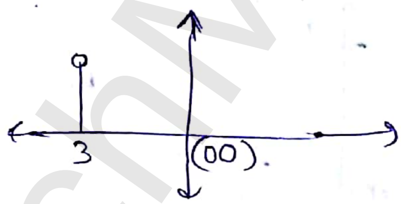
Overall response:

$(\delta(n) * u(n+3)) - (\delta(n-1) * u(n+3))$

$u(n+3) - u[n+3-1]$

$u(n+3) - u[n+2)$

$= \delta(n+3)$



$= \delta(n+3) * \left(\frac{1}{2}\right)^n u(n)$

$= \left(\frac{1}{2}\right)^{n+3} u(n+3)$

Finite convolution duration

Q The i/p to a relaxed LTI system is $x(n) = \{\underset{\uparrow}{1}\ 3\ 3\ 1\}$. the resulting o/p is calculated to be $y(n) = \{\underset{\uparrow}{1}\ 4\ 6\ 4\ 1\}$ determine impulse response of the system.

impr. ***

$x(n) \longrightarrow M$ terms

$h(n) \longrightarrow N$ terms

$y(n) \longrightarrow$ $M+N-1$ terms.

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$x(n) \longrightarrow 4$

$y(n) \longrightarrow 5$

$5 = 4 + N - 1$

$N = 2$

$h(n) = \{a, b\}$

	a	b
1	a	b
3	3a	3b
3	3a	3b
1	a	b

$y(0) = a = 1$

$y(1) = 3a + b \Rightarrow b = 1$

Q. The step response of a LTI system is given by $y_s[n] = \{1, 1, \underset{\uparrow}{2}\}$ what is the o/p of this system when i/p is $x[n] = \{1, \underset{\uparrow}{0}, -1\}$

Dont see this method turn page.

$x(n) \longrightarrow \boxed{H(n)} \longrightarrow y_s[n]$

$x(n) \longrightarrow 3 \quad M$

$y_s(n) \longrightarrow 3$

$3 = \dfrac{M + N - 1}{}$

$3 = 3 + N - 1$

$\boxed{N = 1}$

$H(n) = \{a\}$

	a
1	.
0	
-1	

जो Qus n वाले method से नहीं हो पा रहा है।

$u[n] \longrightarrow \boxed{\quad} \longrightarrow [1, 1, 2]$

$\boxed{H(n) = y_s(n) - y_s[n-1]}$

$= \{1 \; \underset{\uparrow}{1} \; 2\} - \{\underset{\uparrow}{1}, 1, 2\}$

$H(n) = \{1 \; 0 \; 1 \; -2\}$

By Sir

$$x_1(n) = u(n) \xrightarrow{T} y_s(n) = \{1\ \underset{\uparrow}{1}\ 2\}$$

$$x_2(n) = u(n-1) \xrightarrow{T} y_s(n-1) = \{\underset{\uparrow}{1}\ 1\ 2\}$$

$$\delta(n) = u(n) - u(n-1) \xrightarrow{T} y_s(n) - y_s(n-1)$$

$$= \{1\ \underset{\uparrow}{0}\ 1\ -2\} = h(n)$$

	1	0	1	-2
1	1	0	1	-2
0	0	0	0	0
-1	-1	0	-1	2

$$y(n) = \{1, 0, \underset{\uparrow}{0}, -2\ -1\ 2\}$$

Chapter-5

Fourier Series [13 question]

Q1 Consider a continuous time periodic s/g given by $x(t) = 2 + \cos\frac{2\pi}{3}t + 4\sin\frac{5\pi}{3}t$ analyse $x(t)$ and draw its magnitude and phase spectrum.

5th dimension important (*****)

$$x(t) = 2 + \cos\frac{2\pi}{3}t + 4\sin\frac{5\pi}{3}t$$

← this is real s/g, it can be expressed in exponential, but it is not complex s/g

$$\omega_0 = \mathrm{GCD}\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\} = \frac{\pi}{3}$$

$$x(t) = 2\cdot e^{j0} + \frac{1}{2}e^{j2\omega_0 t} + \frac{1}{2}e^{-j2\omega_0 t} + \frac{2}{j}e^{j5\omega_0 t} - \frac{2}{j}e^{-j5\omega_0 t}$$

$a_0 = 2$

$a_2 = \frac{1}{2}$

$a_{-2} = \frac{1}{2}$

$a_5 = \frac{2}{j} = -2j$

$a_{-5} = \frac{-2}{j} = +2j$

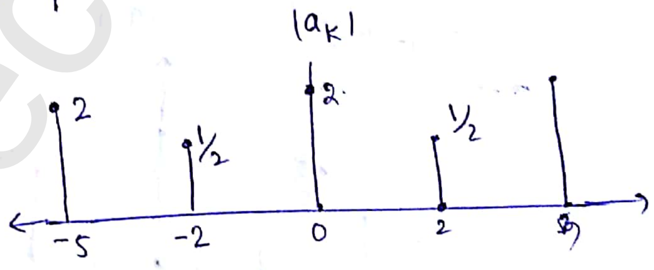
$a_0 = 2 \rightarrow 2\angle 0$

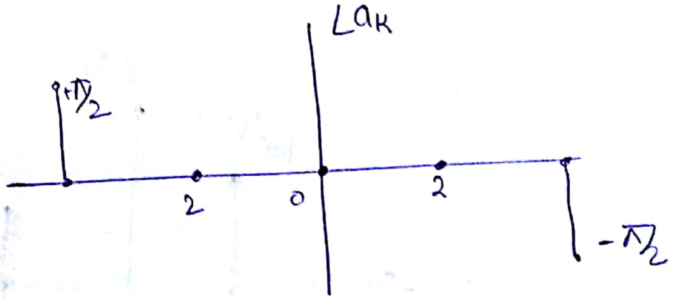
$a_2 = \frac{1}{2} \rightarrow \frac{1}{2}\angle 0$

$a_{-2} = \frac{1}{2} \rightarrow \frac{1}{2}\angle 0$

$a_5 = -2j \rightarrow 2\angle -\pi/2$

$a_{-5} = 2j \rightarrow 2\angle +\pi/2$





Q2 $x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos(2\omega_0 t + \frac{\pi}{4})$

(0) ① ① ② nd Harmonic

1st Harmonic made up of 2 components.

$$x(t) = 1 + \frac{e^{j\omega_0 t}}{2j} - \frac{e^{-j\omega_0 t}}{4j} + e^{j\omega_0 t} + e^{-j\omega_0 t} + \frac{1}{2} e^{j2\omega_0 t} \cdot e^{j\pi/4} + \frac{e^{-j2\omega_0 t}}{2} \cdot e^{-j\pi/4}$$

$= 1$

$a_0 = 1 = 1\angle 0^\circ$

$a_1 = \left(1 + \frac{1}{2j}\right) = \sqrt{1+\frac{1}{4}}\ \angle -\tan^{-1}\frac{1}{2}$

$= \frac{\sqrt{5}}{2}\angle -\tan^{-1}\frac{1}{2}$

$a_{-1} = \left(1 - \frac{1}{2j}\right)$

$= \frac{\sqrt{5}}{2}\angle \tan^{-1}\frac{1}{2}$

$a_2 = \frac{1}{2}e^{j\pi/4}$ $\quad (\cos\pi/4 + j\sin\pi/4) = \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$

$a_2 = \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$

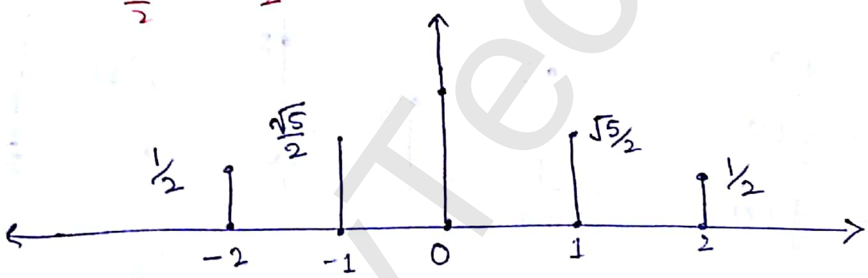
$\sqrt{\frac{1}{8}+\frac{1}{8}} = \sqrt{\ } = \frac{1}{2}\angle \tan^{-1} 1$

$\frac{1}{2}\angle 45^\circ$

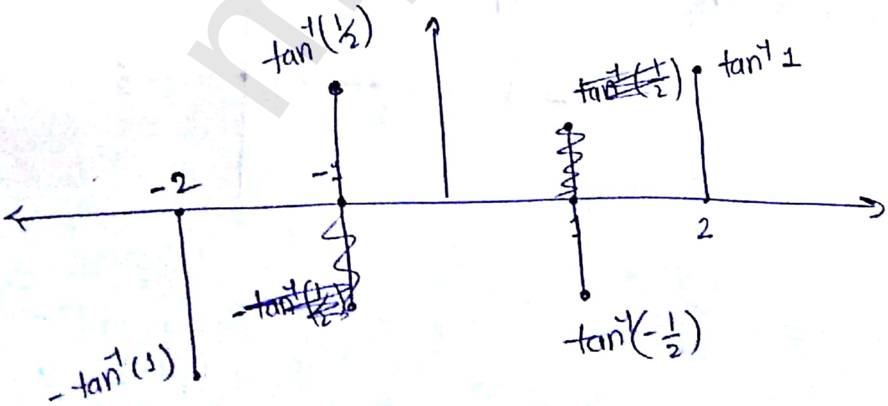
$a_{-2} = \frac{1}{2}e^{-j\pi/4}$

$\frac{1}{2}\angle -45^\circ$

$|a_k|$



$\angle a_k$



Q3 A continuous time periodic signal $x(t)$ has fundamental period $T = 8$ sec and non-zero Fourier coefficients

$a_1 = a_{-1} = 2$

$a_3 = a^*_{-3} = 4j$

Synthesize $x(t)$

Sol^n $T = 8$ sec

$a_1 = a_{-1} = 2$

$a_3 = a_{-3}^* = 4j$

$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/sec

$a_1 = 2 \quad a_{-1} = 2$

$a_3 = 4j$

$a_{-3}^* = 4j$

$a_{-3} = -4j$

ω_0

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k \cdot e^{jk\omega_0 t}$$

$$x(t) = a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_3 e^{j3\omega_0 t} + a_{-3} e^{-j3\omega_0 t}$$

$$= 2 \cdot 2 \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] + 4j \left[\frac{e^{j3\omega_0 t} - e^{-j3\omega_0 t}}{2j}\right] \cdot 2j$$

$$x(t) = 4\cos\omega_0 t - 8\sin 3\omega_0 t$$

$$x(t) = 4\cos\frac{\pi}{4}t - 8\sin 3\frac{\pi}{4}t.$$

Q4 $x(t)$ is periodic with fundamental period $= 4$ sec and $a_k = j \cdot k$ for

$a_k = j \cdot k \quad ; |k| < 3$

$\quad\;\; 0 \quad ;$ otherwise

Synthesize $x(t)$.

$a_0 = 0 \qquad a_2 = 2j$

$a_1 = j \qquad a_{-2} = -2j$

$a_{-1} = -j$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k \cdot e^{jn\omega_0 t} \qquad T = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$= \cancel{a_{-2} e^{-j2\omega_0 t} + a_{-1} e}$$

$$= a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_2 e^{j2\omega_0 t} + a_{-2} e^{-j2\omega_0 t}$$

$$= 2j \cdot j \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \right) + 2j \left(\frac{e^{2j\omega_0 t} - e^{-j2\omega_0 t}}{2j} \right) \cdot 2j$$

$$= -2 \sin \omega_0 t - 4 \sin 2\omega_0 t$$

$$x(t) = -2 \sin \frac{\pi}{2} t - 4 \sin \pi t$$

DO.

Q5 $x(t)$ is periodic with FTP = 2 sec and has the Fourier coefficients given by $a_k = \frac{e^{(2-j2\pi k)} - 1}{1 - j\pi k}$ what is the value of

$$\int_{-1}^{3} x(t)\, dt$$

$$1$$

$$e^2 \cdot e^{-j2\pi k}$$

$$a_0 = \frac{e^2 - 1}{1}$$

$$\cos 2\pi k + j \sin 2\pi k$$

$$a_1 = \frac{e^2 - 1}{1 - j\pi}$$

$$\boxed{a_k = \frac{e^2 - 1}{1 - j\pi k}}$$

$$a_{-1} = \frac{e^2 - 1}{1 + j\pi}$$

By Sir :-

$$a_0 = \frac{e^2 - 1}{1}$$

$$a_0 = \frac{1}{2T}\int_{2T} x(t)\,dt$$

$$2T \cdot a_0 = \int_{2T} x(t)\,dt$$

$$2 \times 2(e^2 - 1) = 4(e^2 - 1)$$

Q.6 The sig $x(t)$ with FTP $= \pi$ is given by $x(t) = \cos t$ in the interval $-\frac{\pi}{2} < t < \frac{\pi}{2}$

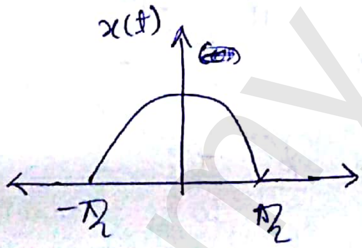
$$x(t) = \cos t \quad ; \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

$$a_k = \frac{1}{\pi} \cdot \frac{\cos \pi k}{1 - 4k^2}$$

another sig $y(t)$ has Fourier cofficient

$$b_k = \frac{(-1)^k}{\pi} \frac{\cos \pi k}{1 - 4k^2}$$

The value of sig $y(t)$ will be.



$$b_k = \frac{(-1)^k}{\pi} \frac{\cos \pi k}{1 - 4k^2} \longrightarrow b_0$$

$$b_k = (-1)^k a_k$$

$$x(t \pm t_0) \xrightarrow{FS} e^{\pm jk\omega_0 t} a_k$$

By Sir

$b_K = (-1)^K a_K$

$(-1)^K = e^{\pm j\pi K}$ } $\rightarrow$ Imp.

$b_K = e^{\pm j\pi K} \cdot a_K$

$\cancel{K}\omega_0 t_0 = \pi \cancel{K}$

$t_0 = \frac{\pi}{2}$

$\omega_0 = \frac{2\pi}{\pi} = 2$

$y(t) = x(t \pm \frac{\pi}{2})$

$y(t) = \cos(t \pm \pi/2)$

$= \sin t$

$= -\sin t$

(11) Do. $b_K = \frac{\cos \pi K}{1-4(K+1)^2} + \frac{\cos \pi K}{1-4(K-1)^2}$ find $y(t)$

$\cos \pi K = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$= \frac{e^{j\pi K} + e^{-j\pi K}}{2}$

$a_K = \frac{1}{\pi} \cdot \frac{\cos \pi K}{1-4K^2}$

$a_{K-1} = \frac{1}{\pi} \frac{\cos \pi(K-1)}{1-4(K-1)^2}$

$= \frac{\cos(\pi K - \pi)}{1-4(K-1)^2} = \frac{1}{\pi}\left(\frac{\cos\pi K \cos\pi + \sin \pi K \sin \pi}{1-4(K-1)^2}\right)$

$a_{K-1} = \frac{1}{\pi} \frac{\cos \pi K}{1-4(K-1)^2}$, $a_{K+1} = \frac{1}{\pi} \cdot \frac{\cos \pi K}{1-4(K+1)^2}$

By Sir.

$$e^{jm\omega_0 t} \cdot x(t) \xleftrightarrow{F.S} a_{k-m}$$

$$e^{-jm\omega_0 t} \cdot x(t) \longleftrightarrow a_{k+m}$$

$$y(t) = -2\pi \cos 2t \cdot x(t)$$

$$\cancel{b_k = \frac{1}{\pi} a_{k-1} + \frac{1}{\pi} a_{k+1}}$$

$$\pi\, a_{k-1} = \frac{1}{\pi} \cdot \frac{1}{\pi} \frac{\cos[\pi(k-1)]}{1-4(k-1)^2}$$

$$\pi\, a_{k+1} = \frac{1}{\pi} \frac{\cos(\pi(k+1))}{1-4(k+1)^2}$$

$$\cancel{-\pi\, a_{k-1} = \cos \pi}$$

$$a_k = \frac{1}{\pi} \frac{\cos \pi k}{1-4k^2}.$$

$$a_{k-1} = \frac{1}{\pi} \frac{\cos \pi(k-1)}{1-4(k-1)^2}$$

$$= \frac{1}{\pi} \frac{\cos \pi k \cos \pi + \sin \pi k \sin \pi}{1-4(k-1)^2}$$

$$a_{k-1} = \frac{-1}{\pi} \frac{\cos \pi k}{1-4(k-1)^2}$$

$$-\pi\, a_{k-1} = \frac{\cos \pi k}{1-4(k-1)^2}$$

$$a_{k+1} = \frac{1}{\pi} \frac{\cos \pi(k+1)}{1-4(k+1)^2}$$

$$= \frac{1}{\pi} \frac{\cos \pi k \cos \pi - \sin \pi k \sin \pi}{1-4(k+1)^2}.$$

$$= \frac{-1}{\pi} \frac{\cos \pi k}{1-4(k+1)^2}$$

$$-\pi\, a_{k+1} = \frac{\cos \pi k}{1-4(k+1)^2}$$

$$b_k = -\pi\, a_{k-1} - \pi\, a_{k+1}$$

$$y(t) = -\pi\, x(t) \cdot e^{+j1\omega_0 t} - \pi\, x(t) \cdot e^{-j\omega_0 t}$$

$$y(t) = -\pi x(t) \cdot \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) \cdot 2$$

$$y(t) = -2\pi x(t) \cdot \cos \omega_0 t$$

$$\boxed{y(t) = -2\pi \cos t \cdot \cos 2t}$$

$$\therefore \omega_0 = \frac{2\pi}{\pi} = 2$$

Q7 The complex exponential fourier series representation fourier series representation of a periodic signal $x(t)$ over fundam. T.P is given by

$$x(t) = \sum_{k=-\infty}^{+\infty} \frac{3}{4+(k\pi)^2} \cdot e^{jk\pi t}$$

. determine the numerical value of A. if one of the component of $x(t)$ is $A\cos 3\pi t$

Soln.

~~$x(t)$~~ =

$$a_k = \frac{3}{4+(k\pi)^2}$$

$$x(t) = \cdots + \frac{A}{2} e^{j3\pi t} + \frac{A}{2} e^{-j3\pi t}$$

$$a_3 = \frac{3}{4+9\pi^2}$$

$$a_{-3} = \frac{3}{4+9\pi^2}$$

$$x(t) = \underline{a_3 e^{j3\omega_0 t} + a_{-3} e^{-j3\omega_0 t}}$$

$$= \frac{3\cdot 2}{4+9\pi^2}\left[\frac{e^{j3\pi t} + e^{-j3\pi t}}{2}\right] \cdot 2$$

$$= \frac{6}{4+9\pi^2} \cdot \cos 3\pi t$$

Time		Freq
Real + E	$\xrightarrow{F.S}$	R + E
R + O	$\longrightarrow$	I + O
I + E	$\longrightarrow$	I + E
I + O	$\longrightarrow$	R + O

Q8 Consider a periodic s/g $x(t)$ given by $x(t) = \sum_{k=-100}^{+100} \cos k\pi \cdot e^{jk\frac{2\pi}{50}\cdot t}$

Tell what type of s/g $x(t)$ is?

$a_k = \cos k\pi \quad ; \quad -100 \leq k \leq 100$

$a_{-k} = \cos(-k\pi) \quad ; \quad -100 \leq k \leq 100$

$= \cos k\pi \quad ; \quad -100 \leq k \leq 100 = a_k \longrightarrow$ even

$a_k^* = \cos k\pi \; ; \; -100 \leq k \leq 100 = a_k \longrightarrow$ real

if $a_k^* = a_k \rightarrow$ real

$a_k^* = -a_k \rightarrow$ img

$x(t)$ will be Even + Real

Q9 $x(t) = \sum_{k=-100}^{+100} j \sin\frac{\pi k}{2} \cdot e^{j\frac{2\pi}{50}k\cdot t}$ will be even or odd

$a_k = j \sin\frac{\pi k}{2}$

$a_{-k} = j \sin\frac{\pi(-k)}{2} = \cancel{-j \sin\frac{\pi k}{2}} \; -100 \leq k \leq 100$

$= -j \sin\frac{\pi k}{2} \quad -100 \leq k \leq 100 \quad = -a_k \rightarrow$ odd.

$a_k^* =$

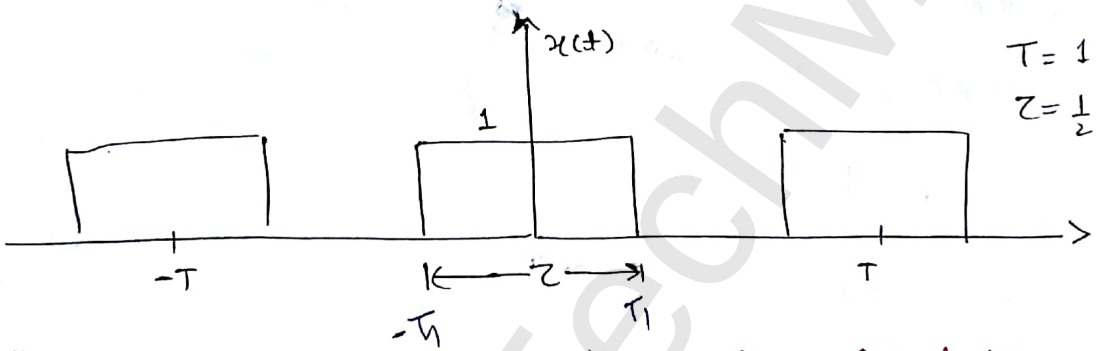
Q. so $x(t) = \sum_{k=-5}^{+5} \sin\frac{2\pi}{3}k \cdot e^{j3\pi k t}$

$a_k = \sin\frac{2\pi}{3}k$ is Real + odd

so $x(t)$ = Imj + odd.

Q 11 The periodic sig shown below is applied as i/p to a filter that cuts off dc as well as freq above 1.2 kHz and produces the o/p y(t). determine ① y(t) (o/p signal)

② what is the power of y(t)



$T = 1$ msec

$\tau = \frac{1}{2}$ msec

Sol^n Remember this the a_k for the periodic pulse train

$$a_k = \frac{\sin k\omega_0 T_1}{k\pi}$$

$$= \quad \omega_0 = \frac{2\pi}{T} = 2\pi = 2\pi \times 1000 = 2000\pi$$

$f = 1000$ Hz.

$$x(t) = \sum_{-\infty}^{+\infty} \frac{\sin k\omega_0 T_1}{k\pi} e^{jn\omega_0 t}$$

$a_0 = 1$

$a_1 = \frac{\sin \omega_0 T_1}{\pi}$ $\qquad a_{-1} = \frac{\sin \omega_0 T_1}{\pi}$

$a_2 = \frac{\sin 2\omega_0 T_1}{2\pi}$ $\qquad a_{-2} = \frac{\sin 2\omega_0 T}{2\pi}$

$a_2 =$

$$x(t) = \frac{\sin \omega_0 T_1}{\pi}\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right].2$$

$$= \frac{2 \sin \omega_0 T_1}{\pi} \cos \omega_0 t$$

$$= 2 \sin \frac{2000\pi}{1000} \times \frac{1}{4} \cos \omega_0 t$$

$$= \frac{2 \sin 2000\pi \times \frac{1}{4\times 1000}}{\pi}$$

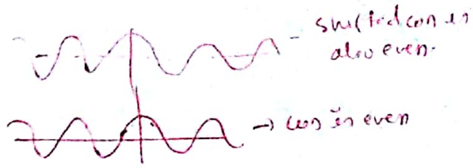
$2T_1 = 8\tau$

$T_1 = \frac{1}{4}$ msec

$$x(t) = \frac{2 \cos 2000\pi t}{\pi}$$

Power of $x(t)$ $\quad P_{x(t)} \quad \frac{\left(\frac{2}{\pi}\right)^2}{2} = \frac{2^2}{\pi^2 . 2} = \frac{2}{\pi^2}$ watt

Symmetry in Fourier Series:-

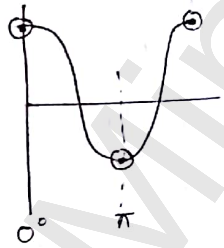


i) Even :- $x(-t) = x(t)$

Magnitude Criterion:- For a pure even symmetric s/g, the dc value may or may not be equal to zero.

Phase Criterion: Since an even symmetric s/g contains only dc and cosine terms so the phase angle at each fn point must be 0° or $\pm\pi$

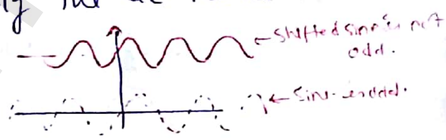
$\angle a_K = 0°$ or $\pm\pi$



ii Odd :- $x(t) = -x(-t)$

a_0 = magnitude criterion- for a pure odd symmetric s/g the dc value must be equal to zero.

$a_0 = 0$



$\angle a_K$

phase criterion : For a pure odd symmetric s/g the phase angle at each fn point must be $\pm\frac{\pi}{2}$ b/c it contains only sin terms

iii) Half wave Symmetry $x(t \pm \frac{T}{2}) = -x(t)$

magnitude criterion

For a pure Half wave symm. s/g the dc and all the even harmonic must be '0'. i.e only odd harmonic can exist.

$a_0 = 0$; $a_K = 0$; K = even

phase criterion : Since both sine and cosine terms can exist simultaneously so there is no phase criterion for pure half wave symmetric signal.

iv) Even + HWS

$$x(-t) = x(t)$$

$$x(t \pm T/2) = -x(t)$$

$\boxed{\cos}$ is a HWS and Even

Magn. criteria :- $a_0 = 0$ $a_k = 0$ even

phase criterion = $\angle a_k = 0^\circ$ or $\pm\pi$

only odd harmonic cosine terms can exist

E+HWS and O+HWS can't be distinguished on basis of magnitude.

(v) Odd + HWS

$$x(-t) = -x(t)$$

$$x(t \mp T/2) = -x(t)$$

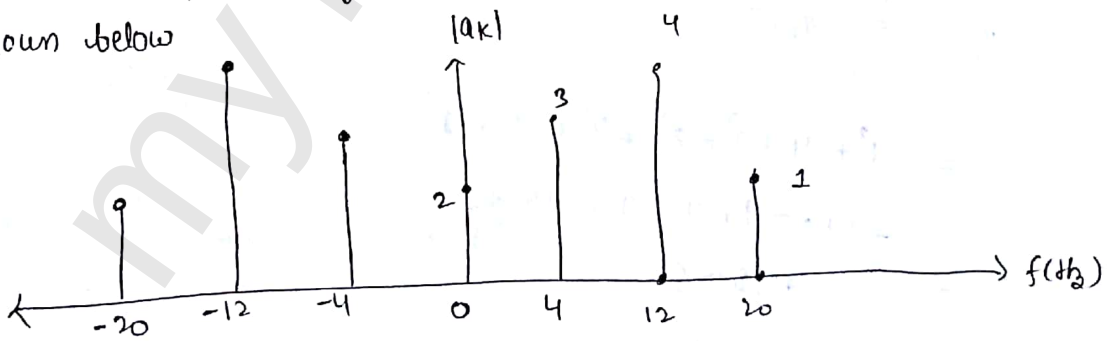
$a_0 = 0$

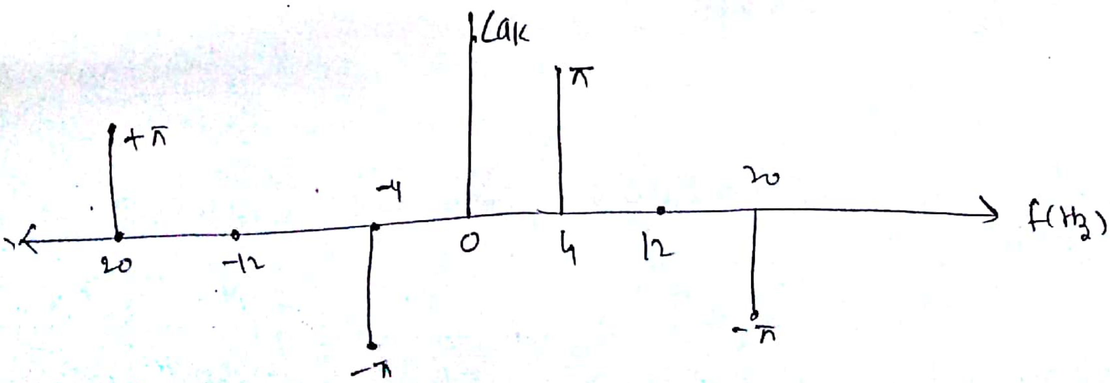
$a_k = 0$; k = even

$\angle a_k = \pm\pi/2$

In odd + HWS s/g [only odd harmonic sin terms can exist]

Q12 Consider a periodic s/g $x(t)$ with magnitude and phase spectrum as shown below





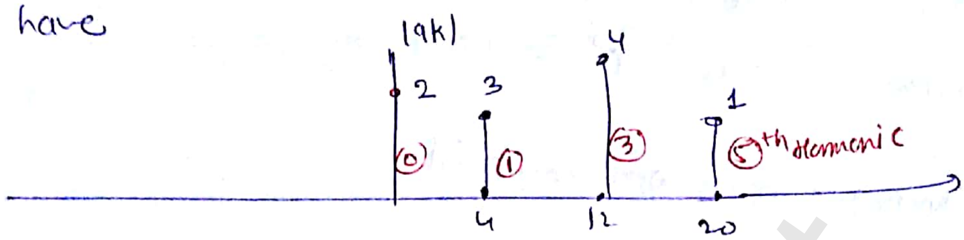
① determine Fundamental freq of sig in radian/sec

GCD of all the individual freq = $\{4, 12, 20\} = 4\ Hz$

Greatest common divisor

Fundamental freq = $4\ Hz$ $\qquad \omega_0 = 8\pi$ rad/sec.

So we have



② determine the type of symmetry present in this periodic sig.

~~Even~~

The signal posses only Even symmetry.

③ determine total average power of the signal

$$P_x = \sum_{-\infty}^{+\infty} |a_k|^2$$

$$= 1^2 + 4^2 + 3^2 + 2^2 + 3^2 + 4^2 + 1^2$$

$$= 1 + 16 + 9 + 4 + 9 + 16 + 1$$

$$= 32 + 18 + 6$$

$$P_x = 56 \text{ watt}$$

④ determine rms value of sig.

$rms = \sqrt{56} = 7.48$

to infy

⑤ Synthesize $x(t)$ ← 1

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k \cdot e^{jk\omega_0 t}$$

$$= a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t}$$

$$a_1 = |a_1| e^{j\angle a_1}$$

$$= 3 \cdot e^{j\pi} \cdot e^{j\omega_0 t} + 3 \cdot e^{-j\pi} \cdot e^{-j\omega_0 t}$$

$$= 3 \left[e^{j(\omega_0 t + \pi)} + e^{-j(3\omega_0 t + \pi)} \right]$$

$$= 6 \cos(\omega_0 t + \pi)$$

$$= -6\cos\omega_0 t$$ ← this is not complete $x(t)$

$$a_3 e^{j3\omega_0 t} + a_{-3} e^{-j3\omega_0 t}$$

$$= \cancel{4[e^{j3\omega_0 t}} = 4\left(\frac{e^{j3\omega_0 t} + e^{-j3\omega_0 t}}{2}\right) \cdot 2$$

$$= 8 \cos 3\omega_0 t$$

$$= a_5 e^{j5\omega_0 t} + a_{-5} e^{-j5\omega_0 t}$$

$$1 \cdot e^{-j\pi} e^{j5\omega_0 t} + 1 \cdot e^{+j\pi} e^{-j5\omega_0 t}$$

$$= \left(\frac{e^{j(5\omega_0 t - \pi)} + e^{-j(5\omega_0 t - \pi)}}{2}\right) \cdot 2$$

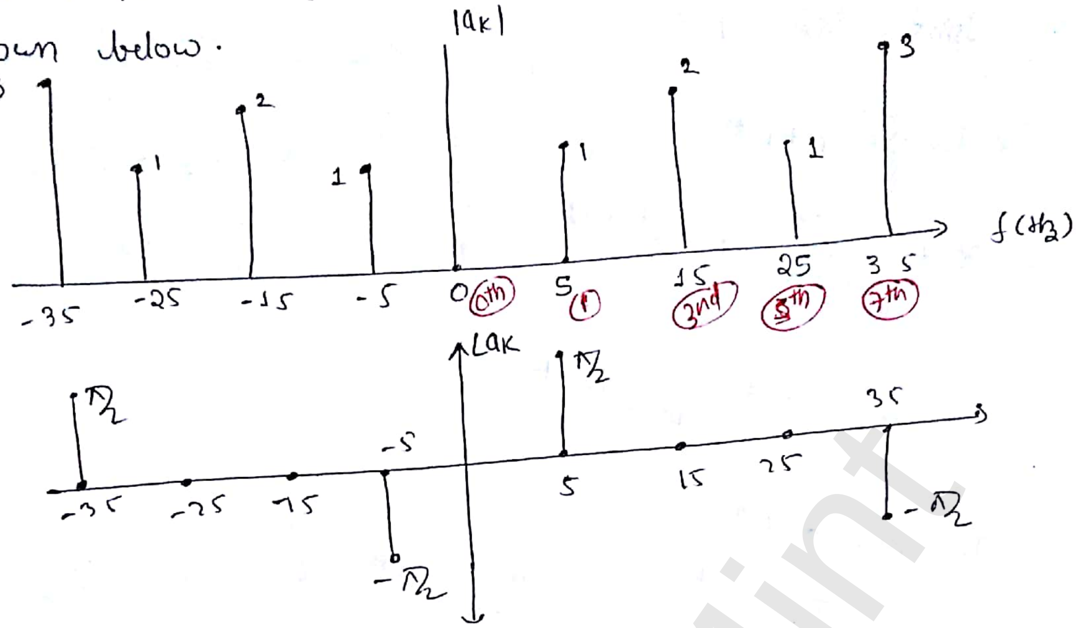
$$= 2 \cdot \underline{\cos(5\omega_0 t - \pi)}$$

$\cos(\theta - \pi) = -\cos\theta$

so $-2\cos 5\omega_0 t$ //

$$x(t) = 2 + (-6\cos\omega_0 t) + 8\cos 3\omega_0 t + 2\cos(5\omega_0 t - \pi)$$

Q13 Consider a periodic sig $x(t)$ with magnitude and phase spectrum as shown below.



$a_0 = 0$, Sin + Cos term.

E ×

O ×

HWS $a_0 = 0$

So signal $x(t)$ is HWS ~~signal~~. symmetry

① fundament frequency

GCD (5, 15, 25, 35)

$f = 5$ Hz

$\omega_0 = 2\pi \cdot 5 = 10\pi$ rad/sec.

② Total power

$= 3^2 + 1^2 + 2^2 + 1^2 + 1^2 + 2^2 + 1^2 + 3^2$

$= 9 + 1 + 4 + 1 + 1 + 4 + 1 + 9$

$= 10 + 10 + 10$

$= 30$ watt

③ RMS

$= \sqrt{30}$

(1) Synthesize $x(t)$

$x(t)$

$$a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t}$$

$$= 1 \cdot e^{j\pi/2} \cdot e^{j\omega_0 t} + e^{-j\pi/2} \cdot e^{-j\omega_0 t}$$

$$= \frac{\left(e^{j(\pi/2+\omega_0 t)} + e^{-j(\pi/2+\omega_0 t)}\right)}{2} \cdot 2$$

$$= 2\cos\left(\frac{\pi}{2}+\omega_0 t\right) = 2\cos\left(\frac{\pi}{2}+10\pi t\right)$$

$$a_3 e^{j3\omega_0 t} + a_3 e^{-j3\omega_0 t}$$

$$= 2 \cdot \frac{\left(e^{j3\omega_0 t} + e^{-j3\omega_0 t}\right)}{2} \cdot 2$$

$$= 4\cos 3\omega_0 t = 4\cos 3 \cdot 10\pi t$$

$$= a_5 e^{j5\omega_0 t} + a_5 e^{-j5\omega_0 t}$$

$$\left(1 \cdot \frac{e^{j5\omega_0 t} + 1 \cdot e^{-j5\omega_0 t}}{2}\right) \cdot 2$$

$$= 2\cos 5\, 10\pi t$$

$$= a_7 \cancel{\cos}\, e^{j7\omega_0 t} + a_{-7} e^{-j7\omega_0 t}$$

$$= 3 \cdot e^{-j\pi/2} \cdot e^{j7\omega_0 t} + 3 \cdot e^{j\pi/2} \cdot e^{-j7\omega_0 t}$$

$$= 3 \cdot e^{j(7\omega_0 t - \pi/2)} + 3 \cdot e^{-j(7\omega_0 t - \pi/2)}$$

$$= 3\left(\frac{e^{j(7\omega_0 t - \pi/2)} + e^{-j(7\omega_0 t - \pi/2)}}{2}\right) \cdot 2$$

$$= 6\cos(7\omega_0 t - \pi/2) = 6\cos\left(7\, 10\pi \cdot t - \frac{\pi}{2}\right)$$

$$x(t) = 2\cos\left(\frac{\pi}{2}+10\pi t\right) + 4\cos 30\pi t + 2\cos 50\pi t + 6\cos(70\pi t - \pi/2)$$

$\downarrow$ $-2\sin(10\pi t)$; $\downarrow$; $\downarrow$; $\downarrow$ $6\sin 70\pi t$

Date - 2 sept

Fourier Transform [13 questions]

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t}\, dt$$

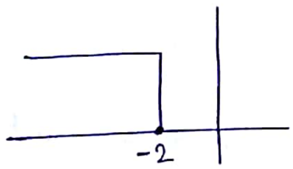
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) \cdot e^{j\omega t}\, d\omega$$

Don't Do
No need.

Q.1 consider a continious time sig $x(t)$ $\xi := \frac{d}{dt}\left[u(-2-t) + u(t-2)\right]$

determine $X(\omega)$.

$u(-2-t)$



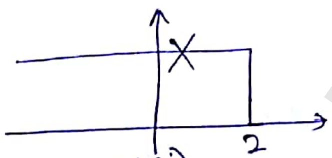
$x(t) = -\delta(t+2) + \delta(t-2)$

$X(\omega) \rightarrow -1 \cdot e^{2j\omega} + 1 \cdot e^{-2j\omega}$

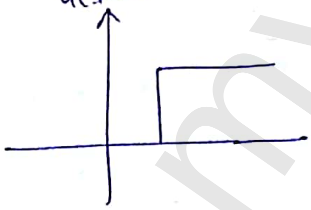
$X(\omega) \rightarrow \frac{-2j(e^{2j\omega} - e^{-2j\omega})}{2j}$

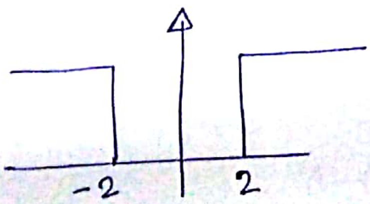
$X(\omega) \rightarrow -2j \sin 2\omega$

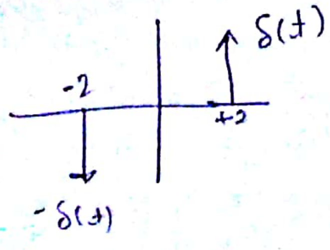
~~u(t-2)~~



$u(t-2)$

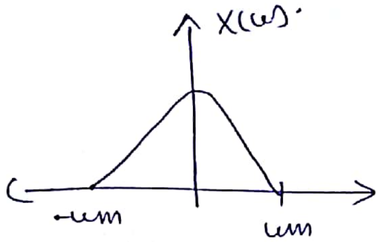






Q.2 let $x(t)$ be a sig such that $X(\omega) = 0$; $|\omega| > \omega_m$
another sig $y(t)$ is specified as having Fourier transform
$Y(\omega) = 2X(\omega - \omega_c)$ determine a sig $m(t)$ such that ~~x(t)~~

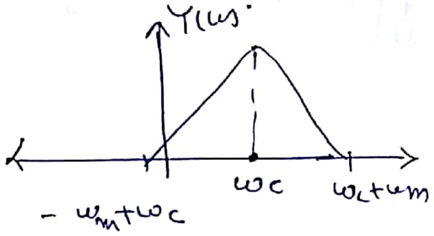
$x(t) = y(t) \cdot m(t)$



$X(\omega) = Y(\omega) * M(\omega)$

$X(\omega) \longrightarrow x(t)$

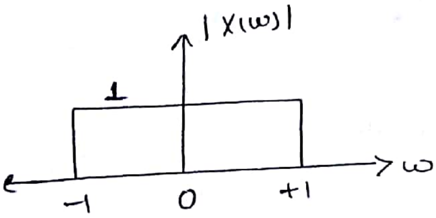
$Y(\omega) \longrightarrow 2x(t) \cdot e^{+j\omega_c t}$

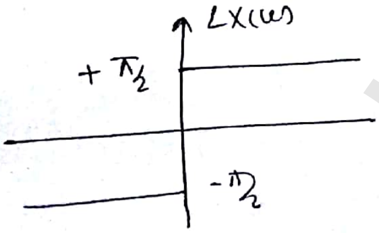


$$\frac{x(t)}{2x(t)\, e^{+j\omega_c t}} = m(t)$$

$$\boxed{\frac{1}{2} e^{-j\omega_c t} = m(t)}$$

** Q.3 consider a continuous time sig $x(t)$ with F.T $X(\omega)$. the magn spectrum as well as the phase spectrum are as shown below. The sig $x(t)$ will be equals to.

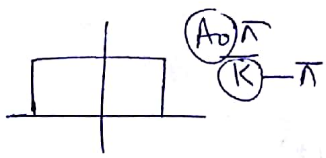


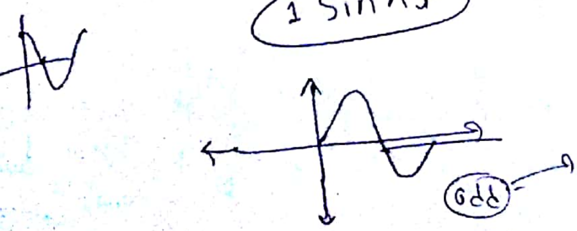


Sol^n

$A_0 \, Sa\, kt$

$1 \sin \pi t$





$$X(\omega) = |X(\omega)|\, e^{j\angle X(\omega)} \qquad e^{j\theta} = \cos\theta + i\sin\theta$$

$$X(\omega) = 1\cdot e^{-j\pi/2} \quad ; \quad -1 \le \omega \le 0$$

$$= 1\cdot e^{j\pi/2} \quad ; \quad 0 < \omega \le 1$$

$$X(\omega) = -j \quad ; \quad -1 \le \omega \le 0$$

$$= j \quad ; \quad 0 \le \omega \le 1$$

$$x(t) = \frac{1}{2\pi}\left[\int_{-1}^{0} -j\cdot e^{j\omega t}\, d\omega + \int_{0}^{1} j e^{j\omega t}\, d\omega\right]$$

$$x(t) = \frac{j}{2\pi}\left[-\frac{e^{j\omega t}}{jt}\Big|_{-1}^{0} + \frac{e^{j\omega t}}{jt}\Big|_{0}^{1}\right]$$

$$x(t) = \frac{j}{2\pi}\left[\frac{-1 + e^{-jt} + e^{jt} - 1}{jt}\right]$$

$$x(t) = \frac{1}{2\pi t}\left[2\cos t - 2\right]$$

$$= \frac{1}{\pi t}\left[\cos t - 1\right]$$

Ans

Q 4 A continuous time sig $x(t)$ has the following spectrum

$$X(\omega) = j\,\delta(\omega-1) + j\delta(\omega-3) + j\delta(\omega) - j\,\delta(\omega+1) - j\,\delta(\omega+3)$$

determine $x(t)$

Soln

$$\delta(t) \longrightarrow 1$$

$$1 \longrightarrow 2\pi\,\delta(\omega)$$

$$\delta(\omega) = \frac{1}{2\pi}$$

$$x(t) = \frac{j}{2\pi}\cdot e^{jt} + \frac{j}{2\pi} e^{3jt} + \frac{j}{2\pi} - \frac{j}{2\pi}\cdot e^{-jt} - \frac{j}{2\pi} e^{-3jt}$$

$$\frac{j}{2\pi} + \frac{2j\cdot j}{2\pi}\left(\frac{e^{jt} - e^{-jt}}{2j}\right) + \frac{2j\cdot j}{2\pi}\left(\frac{e^{3jt} - e^{-3jt}}{2j}\right)$$

$$\frac{j}{2\pi} + \frac{-2\sin t}{2\pi} - \frac{\sin 3t}{\pi}$$

$$= \frac{j}{2\pi} - \frac{\sin t}{\pi} - \frac{\sin 3t}{\pi}$$

$$\sin\omega_0 t \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

$$\delta(t) \longrightarrow 1$$

$$1 \longrightarrow 2\pi\delta(\omega)$$

Q5 Let $X(\omega)$ be the F.T of sig $x(t) = \dfrac{b}{t^2+b}$ where b is a the real value determine the value $\int_{-\infty}^{+\infty} \omega \cdot X(\omega)\, d\omega$

$$x(t) \longrightarrow X(\omega)$$

$$\left(\frac{d}{dt}\right)^n x(t) \longrightarrow (j\omega)^n X(\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\cdot e^{j\omega t}\, d\omega$$

$$\frac{d}{dt}\left(\frac{b}{t^2+b}\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \omega\cdot X(\omega)\cdot e^{j\omega t}\, d\omega$$

$$\frac{-b}{(t^2+b)^2}\cdot 2t \times 2\pi = \int_{-\infty}^{+\infty} \omega\cdot X(\omega)\, d\omega$$

$$\frac{-b \times 2.0}{(0+b)^2} \times 2\pi = 0 = \int_{-\infty}^{+\infty} \omega\cdot X(\omega)\, d\omega.$$

By Sir

$$2\pi\, x(t) = \int_{-\infty}^{+\infty} X(\omega)\cdot e^{j\omega t}\, d\omega$$

$$-j\cdot 2\pi \frac{d}{dt}x(t) = \int_{-\infty}^{+\infty} \omega\cdot X(\omega)\cdot e^{j\omega t}\, d\omega$$

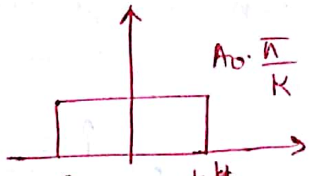
$$-j2\pi \cdot \left.\frac{d\,x(t)}{dt}\right|_{t=0} = \int_{-\infty}^{+\infty} \omega\cdot X(\omega)\, d\omega = 0$$

Q 8 Consider a continuous time sig $x(t)$ given by

$$x(t) = \int_{-\infty}^{+\infty} \frac{\sin 3\tau}{\tau} \cdot \frac{\sin(t-\tau)}{(t-\tau)}\, d\tau$$

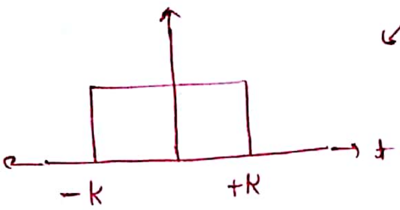
simplify $x(t)$

$A_0\, Sa(Kt) \longrightarrow$



$A_0 \cdot \frac{\pi}{K}$

$-a$, $-K$ $+K$



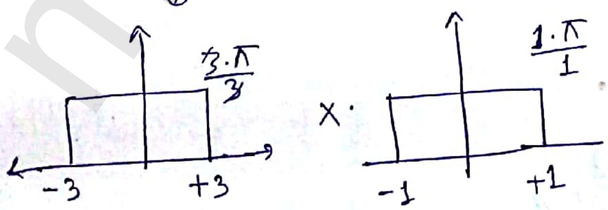
$2\pi A_0\, Sa(K\omega)$

3 Sa 3t

$$x(t) = \int_{-\infty}^{+\infty} \frac{\sin 3\tau}{\tau} \cdot \frac{\sin(t-\tau)}{(t-\tau)}$$

$$3 \cdot \frac{\sin 3t}{3t} \qquad \frac{\sin t}{t}$$

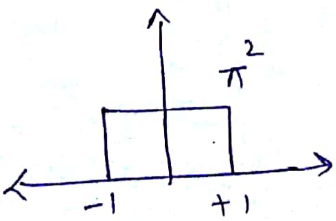
$$3 \cdot Sa\, 3t \quad * \quad Sa\, t$$



$\frac{3 \cdot \pi}{3}$ $\times$ $\frac{1 \cdot \pi}{1}$

$$\pi^2 = A_0 \cdot \frac{\pi}{1}$$

$X(\omega) \rightarrow$



$\xrightarrow{x(t)}$

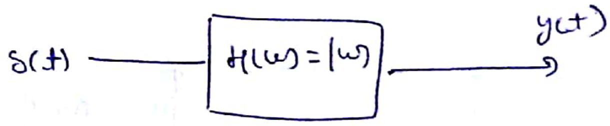
$\pi\, Sa(t)$

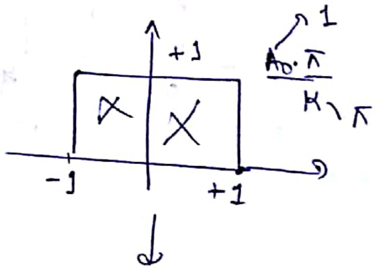
$$\frac{\pi \sin t}{t} = \frac{\pi \sin t}{t} \quad //$$

Q.7 An impulse signal $\delta(t)$ is sent as i/p to a continuous time LTI system whose freq^n response is $H(\omega) = |\omega|$; $|\omega| < 1$

; 0 otherwise.

determine the energy of o/p sig $y(t)$



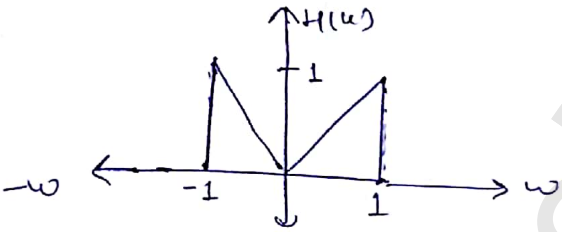


$A_0 \cdot \pi \rightarrow 1$

$K \cdot \pi$

$\frac{1}{\pi} \cdot Sa(\pi t)$

$h(t) = Sa(\pi t)$



$y(t) — Y(\omega) \rightarrow H(\omega)$

$$E = \int_{-\infty}^{+\infty} y(t)^2 \, dt$$

$$\boxed{E = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |Y(\omega)|^2 \, d\omega}$$

$$E = \frac{1}{2\pi} \times 2 \times \int_0^1 \omega^2 \, d\omega$$

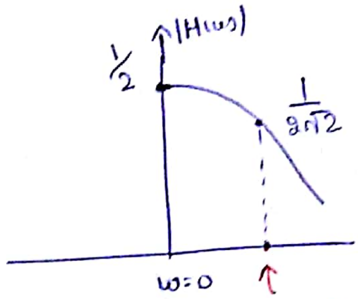
$$E = \frac{1}{\pi} \left[\frac{\omega^3}{3}\right]_0^1 = \frac{1}{3\pi}$$

Q.8 Consider a low pass non causal LTI system with impulse respon.

$h(t) = e^{2t}u(-t)$. The 3 dB bandwidth for this LTI system will be.

Solⁿ $H(\omega) = \frac{-1}{j\omega - 2} = \frac{1}{2 - j\omega}$

at $\omega = 0$ the magnitude is max^m.



3dB freq' point → The point at which magnitude becomes $\frac{1}{\sqrt{2}}$ times of max^m magnitude and The power becomes half.

Power becomes half means power reduces by 3dB.

$$\frac{1}{\sqrt{2^2 + \omega^2}} = \frac{1}{2\sqrt{2}}$$

$$\frac{1}{2^2 + \omega^2} = \frac{1}{4 \cdot 2}$$

$$4 \cdot 2 = 4 + \omega^2$$

$$8 - 4 = \omega^2$$

$$\omega = 2 \text{ rad/sec}$$

By Sir $H(\omega) = \frac{1}{2 - j\omega}$

$$|H(\omega)|_{\omega=0} = \frac{1}{\sqrt{\omega^2 + 4}} = \frac{1}{2}$$

$$|H(\omega)|_{\omega = \omega_{3dB}} = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{4 + \omega_{3dB}^2}} = \frac{1}{2\sqrt{2}}$$

$$\omega_{3dB} = 2 \text{ rad/sec}$$

Q.9 Consider a continious time sig ~~x(t) = 2. Sin[2π 10(t − 1/100)]~~

$$x(t) = \frac{2\sin[2\pi\cdot 10\,(t - 1/100)]}{\pi\cdot(t - 1/100)}\cdot\cos 2\pi\cdot 100\,t$$

the Energy of sig x(t) will be

$$x(t) = \frac{\frac{2\cdot 10\cdot 2}{10}\,\sin[2\pi\cdot 10\,(t - 1/100)]}{2\cdot 10\cdot\pi\,(t - 1/100)}$$

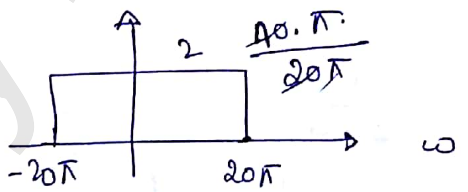
t' t'

$$40\ \frac{Sa[\ldots t]}{\vdots\ \cancel{t}}\cdot\cos\omega_0 t$$

$$E = \int_{-\infty}^{+\infty} x(t)^2\,dt$$

$E =$

$\cos\omega_0 t$ $\quad 40\cdot Sa[\ t]$



$$2\sin\left[2\pi\,10\,t - \frac{2\pi 10}{100}\right]$$

$$\sin 2\pi 10 t\,\cos\frac{2\pi 10}{100} - \cos 2\pi 10 t\,\sin\frac{2\pi 10}{100}$$

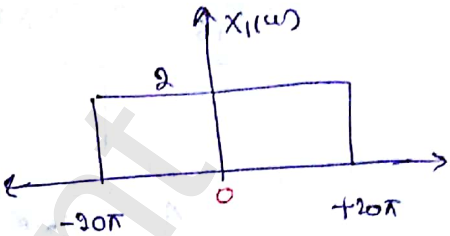
By Sir

$$x(t) = 2 \cdot \frac{\sin\left[2\pi\, 10(t)\right]}{\pi(t)} \cdot \cos 2\pi\, 100t$$

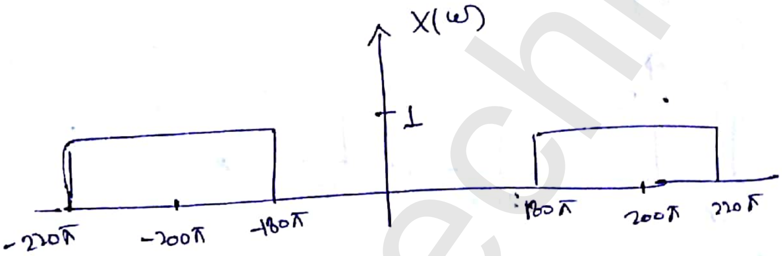
$$E_x = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2 \, d\omega$$

$$x(t) = x_1(t)\cdot\cos 200\pi t \xleftrightarrow{F.T} \frac{1}{2}\left[X_1(\omega-200\pi) + X_1(\omega+200\pi)\right]$$

$$x_1(t) = \frac{2\sin 20\pi t}{\pi t} \xleftrightarrow{F.T}$$



$$x(t)\cdot\cos\omega_0 t \xleftrightarrow{F.T} \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$



$$E_x = \frac{1}{2\pi}\cdot 2\cdot 40\pi = 40\ J$$

Q.10

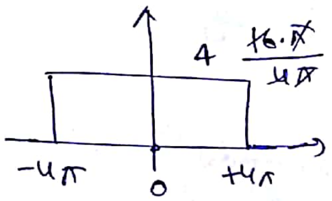
$$x(t) = \frac{4 \cdot \sin 2\pi\, 2\left(t - \frac{1}{40}\right)}{\pi\left(t - \frac{1}{40}\right)} \cdot \cos 2\pi 12 t$$

Find Energy E = ?

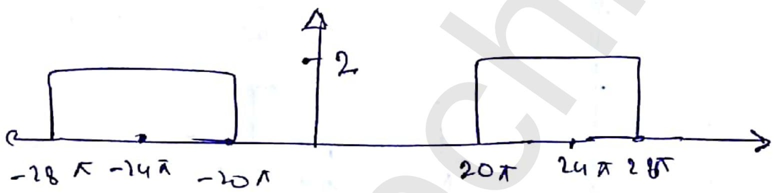
$$= 4 \cdot 4 \, \frac{Sa(2\pi\, 2t)}{2\pi\, \pi t} \cdot \cos 2\pi 12 t$$

$$= \underbrace{16 \cdot Sa(4\pi t)} \cdot \cos 2\pi 12 t$$

$\downarrow$



$\downarrow$



$$E = \frac{1}{2\pi} \int_{-\infty}^{+\infty} 2 \times 4 \times 8\pi$$

$$E = \frac{6 \times 8\pi}{2\pi} = \frac{32\pi}{\pi} = 32 \text{ Joules}$$

Most impr

Response of LTI System to complex Exponential :

$$x(t) = A.e^{j\omega_0 t} \xrightarrow[h(t)]{} y(t) = A\cdot e^{j\omega_0 t} \cdot H(\omega_0)$$

$H(\omega)$

$H(\omega)\Big|_{\omega=\omega_0}$ → impulse response at $\omega=\omega_0$

Q11 consider a continious time LTI system with impulse response $h(t)=3$ ~~for~~ $h(t) = 3 \; ; \; 0 \le t \le 3$

$0 \; ;$ otherwise

the system is supplied with a constant i/p $x(t) = 5$, the steady state o/p $y(t)$ will be.

Derivation

$$x(t) = Ae^{j\omega_0 t} \xrightarrow[h(t)]{} y(t) = x(t)*h(t) = h(t)*x(t)$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$x(t-\tau) = A.e^{j\omega_0(t-\tau)}$$

$$= A e^{j\omega_0 t} \cdot e^{-j\omega_0\tau}$$

$$= \int_{-\infty}^{+\infty} h(\tau)\, A.e^{j\omega_0 t} \cdot e^{-j\omega_0\tau} d\tau$$

$$y(t) = A\cdot e^{j\omega_0 t} \cdot \underbrace{\int_{-\infty}^{+\infty} h(\tau)\cdot e^{-j\omega_0\tau} d\tau}$$

$$= A\cdot e^{j\omega_0 t} \; H(\omega)\Big|_{\omega=\omega_0}.$$

Soln 11

$$x(t) = 5 = 5\cdot e^{j0} \xrightarrow{} y(t) = 5\cdot e^{j0}\cdot H(0)$$

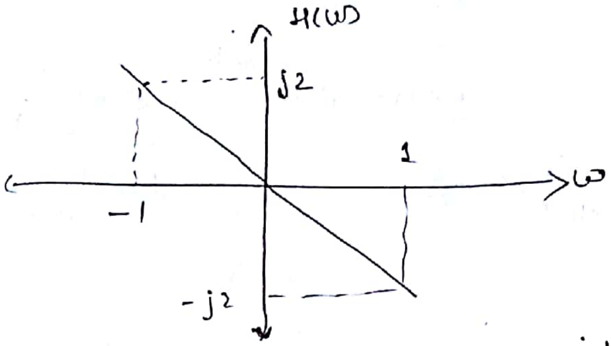
$$H(\omega) = \int_{-\infty}^{+\infty} h(t)\cdot e^{-j\omega t} dt$$

$$H(0) = \int_{-\infty}^{+\infty} h(t)\cdot e\, dt$$

$$H(0) = 9$$

$$y(t) = 5\cdot e^{j0}\cdot 9 = 45$$

Q12 A causal LTI system has a freq Response $H(\omega)$ as shown below



if the i/p applied to the system is $x(t) = e^{-jt}$ then the value of o/p $y(t)$ will be

$e^{-\frac{\pi}{4}t}$

Soln $e^{-jt} = x(t) \xrightarrow[h(t)]{} y(t) = e^{-jt} H(-1)$

$y(t) = e^{-jt} \cdot 2j$

Q13 The o/p $y(t)$ of a particular continuous time LTI system is given by $y(t) = \int_{t-1}^{t} x(\tau)\,d\tau$ determine

① impulse response of the system $h(t)$

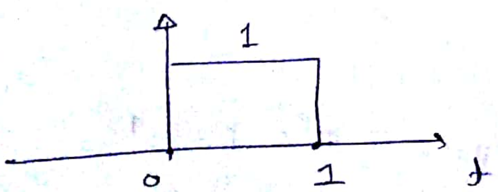
② determine o/p $y(t)$ if the i/p to the system is $x(t) = \cos \pi t + \sin(2\pi t + \frac{\pi}{4})$

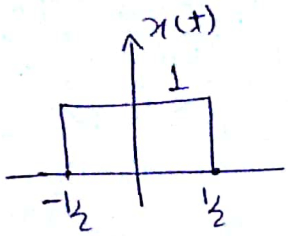
Soln 13 $y(t) = \int_{t-1}^{t} x(\tau)\,d\tau$

$h(t) = y(t) = \int_{t-1}^{t} \delta(\tau)\,d\tau$

$h(t) = u(t) - u(t-1)$

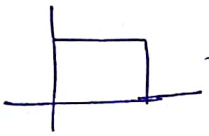
$h(t) =$





$\longrightarrow \quad 1 \cdot Sa\left(\frac{\omega}{2}\right)$

$x(t-\frac{1}{2}) \longrightarrow \quad Sa\left(\frac{\omega}{2}\right) e^{-j\omega \frac{1}{2}}$



$\longrightarrow \quad Sa\left(\frac{\omega}{2}\right) e^{-\frac{j\omega}{2}} = \frac{2}{\omega} \sin\left(\frac{\omega}{2}\right) \quad e^{-\frac{j\omega}{2}}$

$x(t) = \cos \pi t + \sin(2\pi t + \frac{\pi}{4})$

$$x(t) = \frac{e^{i\pi t} + e^{-i\pi t}}{2} + \frac{e^{i(2\pi t + \pi/4)} - e^{i(2\pi t + \pi/4)}}{2j}$$

$$x(t) = \underset{\uparrow \text{①}}{\frac{e^{i\pi t}}{2}} + \underset{\uparrow \text{②}}{\frac{e^{-i\pi t}}{2}} + \underset{\uparrow \text{③}}{\frac{e^{i2\pi t}}{2j} \cdot e^{i\pi/4}} - \underset{\uparrow \text{④}}{\frac{e^{i2\pi t}}{2j} \cdot e^{i\pi/4}}$$

$$x(t) \xrightarrow{h(t)} y(t) \rightarrow \frac{e^{i\pi t}}{2} \cdot \frac{2}{\pi} \cdot e^{-j\pi/2}$$

$$\frac{e^{-i\pi t}}{2} \cdot \frac{2}{-\pi} \cdot (-1) \cdot e^{j\pi/2}$$

$$\frac{e^{i\pi/4}}{2j} \cdot e^{i2\pi t} \quad \frac{2}{2\pi} \sin \frac{2\pi}{2} \quad e^{-j2\pi/2} \rightarrow 0$$

$\rightarrow 0$

$\frac{2}{\pi} \cos(\pi t - \frac{\pi}{2})$

$\frac{2}{\pi} \sin \pi t$ // Answer

$$= \frac{2}{\pi}\left\{ \frac{e^{i\pi t}}{2} e^{-j\pi/2} + \frac{e^{-i\pi t}}{2} \cdot e^{j\pi/2} \right\}$$

$$= \frac{1}{\pi}\left\{ e^{i(\pi t - \pi/2)} + e^{i(-\pi t + \pi/2)} \right\} = \frac{2}{\pi}\left\{ \frac{e^{i\theta} + e^{-i\theta}}{2} \right\}$$

By Sir

$$h(t) = u(t) - u(t-1)$$

$$H(\omega) = \frac{1}{j\omega} + \pi \cancel{\delta(\omega)} - e^{-j\omega}\left[\frac{1}{j\omega} + \pi \cancel{\delta(\omega)}\right]$$

$$H(\omega) = \frac{1 - e^{-j\omega}}{j\omega}$$

$$\sin\left(2\pi t + \frac{\pi}{4}\right) = \frac{1}{2j} e^{j(2\pi t + \pi/4)} - \frac{1}{2j} e^{-j(2\pi t + \pi/4)}$$

$$= \frac{1}{2j} e^{j(2\pi t + \pi/4)} \xrightarrow{\;T\;} 0$$

$$\cos \pi t \longrightarrow \frac{1}{2} e^{j\pi t} + \frac{1}{2} e^{-j\pi t}$$

$$\frac{1}{2} e^{j\pi t} \xrightarrow{\;T\;} \frac{1}{2} e^{j\pi t} \cdot \frac{2}{j\pi}$$

$$\frac{1}{2} e^{-j\pi t} \xrightarrow{\;T\;} \frac{1}{2} e^{-j\pi t} \frac{2}{-j\pi}$$

$$= \frac{2j}{j\pi}\left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right]$$

$$= \frac{2}{\pi} \sin \pi t$$

Sampling Theorem [Sampling]

$x(t) \rightarrow \otimes \rightarrow y(t) = x(t) \cdot p(t) \xleftrightarrow{FT} \frac{1}{2\pi}\left[X(\omega) * P(\omega)\right]$

$p(t) = \sum_{-\infty}^{+\infty} \delta(t - KT_s)$

$$Y(\omega) = \frac{1}{2\pi}\left[X(\omega) * \frac{2\pi}{T_s}\sum_{k=-\infty}^{+\infty}\delta(\omega - K\omega_s)\right]$$

Recall $\rightarrow$ $P(\omega) = \frac{2\pi}{T_s}\sum_{-\infty}^{+\infty}\delta(\omega - K\omega_s)$

$$Y(\omega) = \frac{1}{T_s}\sum_{K=-\infty}^{+\infty}\left[X(\omega) * \delta(\omega - K\omega_s)\right]$$

$$\boxed{Y(\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{+\infty} X(\omega - K\omega_s)}$$

$\rightarrow$ min^m fn^c of sig.

Nyquist rate = $\omega_s(\text{min}) = 2\,\omega_m$

Nyquist interval = $(T_s)_{max} = \frac{2\pi}{\omega_s(\text{min})}$

Q.1 let $x(t) = \text{Sinc}\ 400t + \text{Sinc}\ 1200t$ determine its nyquist rate.

$$\frac{\sin 400\pi t}{400\pi t} + \frac{\sin \boxed{1200\pi} t}{1200\pi t}$$

$\Downarrow$ max^m fn^c = 400π r/sec

$\Downarrow$ max^n fn' : 1200π r/sec

$\omega_s = 2400\pi$ rad/sec

Q.2 $x(t) = \text{sinc}^2\, 500t + \text{sinc}\, 900t$

$$\frac{\sin^2 500\pi t}{(500\pi t)^2} + \frac{\sin 900\pi t}{900\pi t}$$

$\Downarrow$ $\Downarrow$

$\max^m f_m^c = 1000\pi$ r/s 900π r/s

$N.R = 2000\pi$ r/sec

2007 Gate EC Q3

$x(t) = \text{sinc}\, 300t + \text{sinc}\, 700t$

$$= \frac{\sin \pi 300t}{300\pi t} + \frac{\sin 700\pi t}{700\pi t}$$

$\Downarrow$ $\Downarrow$

300π 700π

$\omega_{s(min)} = 1400\pi$

$$T_s = \frac{2\pi}{1400\pi} = \frac{1}{700}\ \text{sec}$$

Q.4 $x(t) = \text{sinc}^2\, 800t + \text{sinc}\, 1400t$

$$\frac{\sin^2 800\pi t}{800\pi t} + \frac{\sin^2 1400\pi t}{1400\pi t}$$

$\Downarrow$ $\Downarrow$

1600π 1400π

$(\omega_s)_{min}$ 3200π r/sec = Ωm

$$(T_s)_{max} = \frac{2\pi}{3200\pi} = \frac{1}{1600}\ \text{sec.}$$

Q 5 Consider a continuous time sig $x(t)$ with Nyquist rate ω_0. determine the nyquist rate for each and every following sig

i) $x_1(t) = x(t-t_0) + x(t+t_0)$

ii) $x_2(t) = \frac{d}{dt}x(t)$

iii) $x_3(t) = x^2(t)$

iv) $x_4(t) = x(t)\cdot\cos\omega_0 t$

v) $x_5(t) = x(2t)$

vi) $x_6(t) = x\left(\frac{t}{2}\right)$

Soln $x(t) \longrightarrow \omega m$

~~(1) ω_0~~

(2) ω_0

(3) $\cancel{2(\omega_0+\omega_0)}$

(4)

By

(1) $N\cdot R = \omega_0$

$$\text{so } \omega m = \frac{\omega_0}{2}$$

$$x_1(t) = \underset{\Downarrow}{x(t-t_0)} + \underset{\Downarrow}{x(t+t_0)}$$

$$\frac{\omega_0}{2} \qquad\qquad \frac{\omega_0}{2}$$

$$N\cdot R = 2\times\frac{\omega_0}{2} = \omega_0$$

(2) $N\cdot R_2 = \omega_0$

(3) $x_3(t) = x^2(t)$

$$= \frac{\omega_0}{2} + \frac{\omega_0}{2} = \omega_0$$

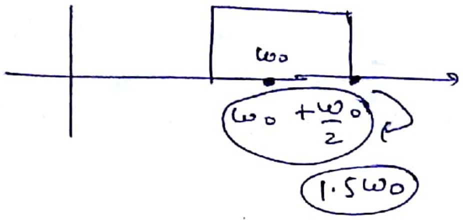
$$NR = 2\omega_0$$

iv) $x_4(t) = x(t) \cdot \cos\omega_0 t$

$\Downarrow$ $\Downarrow$

max^m frq^c $\frac{\omega_0}{2}$ $+ \; \omega_0$ max^m frq^c $= \frac{3}{2}\omega_0$ ← Total max^m frq^c

$NR = 3\omega_0$



(v) $x_5(t) = x(2t)$

compression in time by 2 = expansion in frq^c by 2.

max^m frq^c → $\omega_m = \frac{\omega_0}{2}$ for $x(t)$

max^m frq^c of $x(2t) = 2 \times \frac{\omega_0}{2}$

$\omega_m = \omega_0$

so $NR = 2\omega_0$

vi) $x_6(t) = x(t/2)$

$\Downarrow$

max^m frq^c $\frac{\omega_0}{4}$ $NR = \frac{\omega_0}{2}$

Q.6 A sig given by $x(t) = 5\cos 400\pi t$ is sampled at a rate of 300 samples/sec. the resulting samples are passed through an ideal low pass filter with cut-off frq^c 150 Hz. what are the frq^cies that will be present at the o/p of low pass filter.

$$x(t) = 5\cos 400\pi t \xrightarrow{F.T} 5\pi\left[\delta(\omega - 400\pi) + \delta(\omega + 400\pi)\right]$$

$f_s = 300$ samples/sec

$\omega_s = 600\pi$ rad/sec

$f_c = 150\,Hz$

$\omega_c = 300\pi$

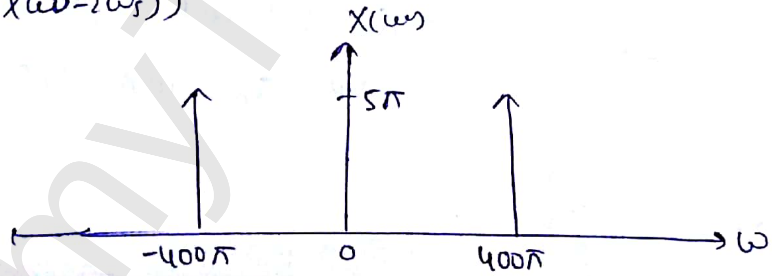
$$Y(\omega) = \frac{1}{T_s}\sum_{-\infty}^{+\infty} X(\omega - k\omega_s)$$

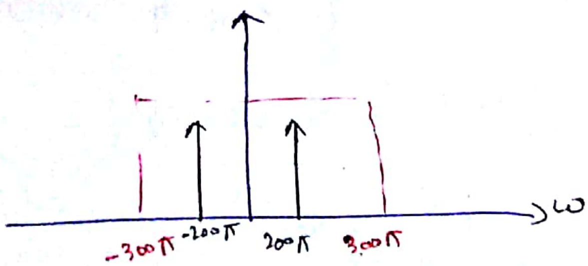
$k=0$; $\frac{1}{T_s} \cdot X(\omega)$

$k=1$; $\frac{1}{T_s} \cdot X(\omega - \omega_s)$

$k=2$; $\frac{1}{T_s}(X(\omega - 2\omega_s))$

$k=-1$; $\frac{1}{T_s} X(\omega + \omega_s)$





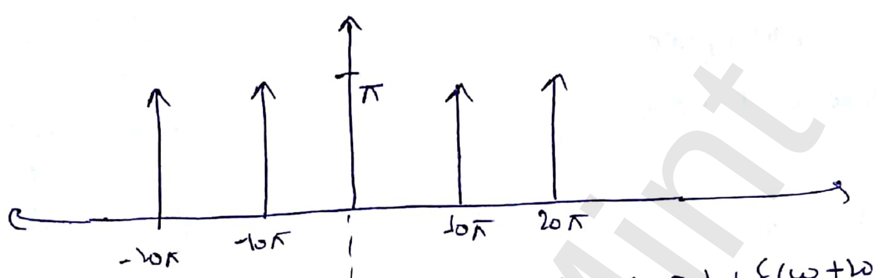
$\omega = 200\pi$ rad/sec

Q7 Consider a continuous time sig $x(t)$ given by

$$x(t) = \cos 10\pi t + \cos 20\pi t$$

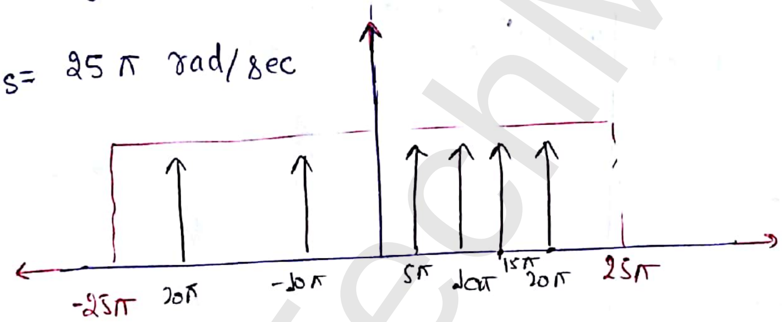
the sig is sampled at a rate of 25π rad/sec and the sampled sig is passed through an ideal low pass filter with cut off freq 25π rad/sec. The no. of signals and corresponding frequencies that will be present at the o/p will be.

Solⁿ



$$X(\omega) = \pi[\delta(\omega - 10\pi) + \delta(\omega+10\pi) + \delta(\omega-20\pi) + \delta(\omega+20\pi)]$$

$\omega_s = 25\pi$ rad/sec



$$Y(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s)$$

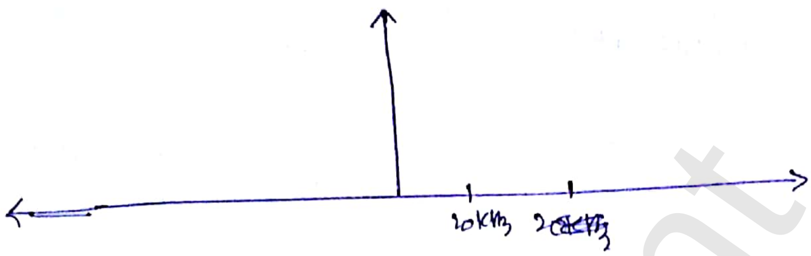
Four freq will be generate at o/p. 5π, 10π, 15π, 20π rad/sec

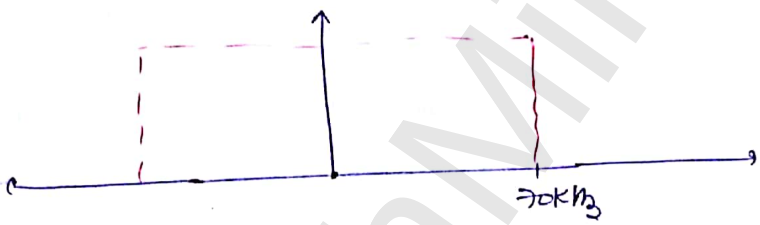
$k=0$; $\frac{1}{T_s} X(\omega)$

$k=1$; $\frac{1}{T_s} X(\omega - 25\pi)$

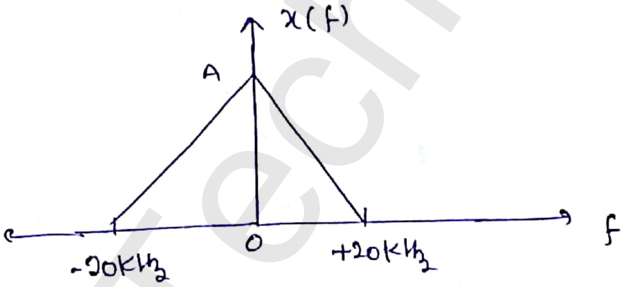
Q.8 In A Base band s/g $x(t)$ has BW 20 kHz. The s/g is sampled at a rate of f_s and passed through an ideal low pass filter with cut off f_q^c 70 kHz to recover the original s/g. What should be the min^m sampling f_q^c to avoid the distortion

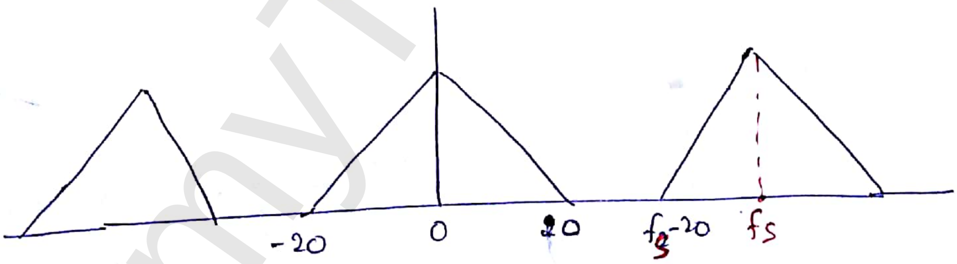
Sol^n For a Base band s/g the bandwidth = max^m cut off f_q^c





By Sir





$f_s - 20 \geq f_c$

$\therefore f_{s\,min} \geq f_c + 20$

$= 90\ \text{kHz}$

Laplace Transform [12 questions]

$$X(s) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st}\, dt$$

$e^{-at} u(t) \longrightarrow \frac{1}{s+a}$; $Re[s] > -a$ — ROC only depend on Real part.

$-e^{-at} u(-t) \longrightarrow \frac{1}{s+a}$; $Re[s] < -a$

IVT $\rightarrow$ for causal s/g and system.

FVT $\rightarrow$ Poles at LHS with atmost pole at origin.

Q1 Consider a continuous time s/g $x(t) = e^{-10t} u(t-4)$.
another s/g $g(t) = A e^{-10t} u(-t-t_0)$.
determine the values of const A and t_0 such that Laplace of $g(t)$ and $x(t)$ have same algebraic form.

$u(t) \longrightarrow \frac{1}{s}$

$$X(s) = \frac{1}{s+10} \cdot e^{-4s}$$

$$G(s) = \frac{-A}{s+10} e^{-st_0}$$

$-A = 1$ $\quad\quad$ $-4s = -st_0$

$\boxed{A = -1}$ $\quad\quad$ $\boxed{t_0 = 4}$

$e^{-10(t-4+4)} \cdot u(t-4)$

$x(t) = e^{-10t} u(t-4)$ $\qquad u(t)$

$x(t) = e^{-10(t-4+4)} u(t-4)$

$= \underbrace{e^{-10(t-4)} \cdot e^{-40} u(t-4)}$

By Sir

$x(t) = e^{-10t} u(t-4)$

$u(t-4) \rightarrow \frac{e^{-4s}}{s}$

$X(s+10) \longrightarrow \frac{e^{-4(s+10)}}{s+10}$

$g(t) = A \cdot e^{-10t} u(-t-t_0)$

$u(t-t_0) \rightarrow \frac{e^{-st_0}}{s}$ ← Imp^r step.

$u(-t-t_0) \rightarrow -\frac{e^{st_0}}{s}$

$X(s+10) = -\frac{e^{(s+10)t_0}}{s+10}$

$\rightarrow -A \cdot \frac{e^{(s+10)t_0}}{(s+10)}$

$-A = 1 \qquad t_0 = -4$

$\boxed{A = -1} \qquad \boxed{t_0 = -4}$

***

$u(t) = \frac{1}{s} \qquad u(-t) = -\frac{1}{s}$

$u(t-4) = \frac{e^{-4s}}{s} \qquad u(-t-t_0) = -\frac{e^{st_0}}{s}$

$u(t+4) = \frac{e^{+4s}}{s} \qquad u(-t+t_0) = -\frac{e^{-st_0}}{s}$

Q 2 The laplace transform of a continious time sig $x(t)$ is given by

$$X(s) = \frac{5-s}{s^2-s-2}.$$

if the Fourier transform of this sig exist then the value of sig $x(t)$ will be.

$$\frac{+1 \pm \sqrt{1+8}}{2} \qquad \frac{1 \pm 3}{2} \qquad \frac{1+3}{2}, \frac{1-3}{2}$$

$$2, -1$$

$$(s-2) \quad (s+1)$$

$$\frac{5-s}{(s-2)(s+1)}$$

$$= \frac{A}{s-2} + \frac{B}{(s+1)} \qquad \frac{6}{-3}$$

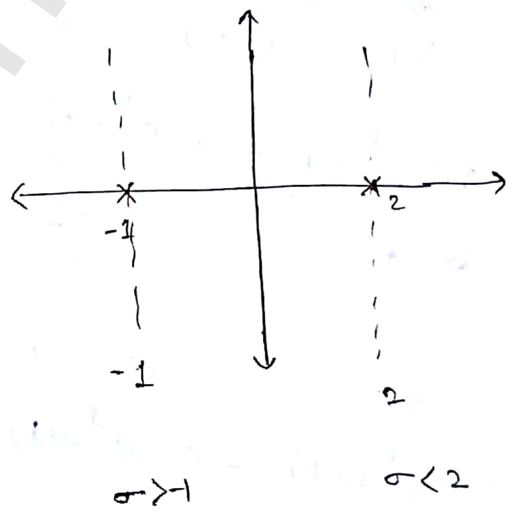
$$\frac{1}{s-2} - \frac{2}{(s+1)}$$

$$e^{2t}u(t) \quad - 2e^{-t}u(t)$$

$$\sigma > 2 \qquad \sigma > -1$$

$$x(t) = -e^{2t}u(-t) - 2e^{-t}u(t)$$

$$\sigma < 2 \qquad \sigma > -1$$



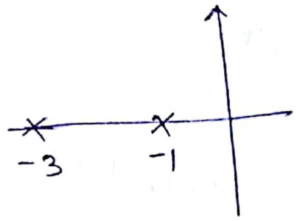
For General sigs if ROC include jω axis sig stable.

For causal sigs the poles should be in left side.

Q3 Let $x(t)$ be a sig that has a rational Laplace transform with exactly 2 poles located at $s=-1$ and $s=-3$. if

$$g(t) = e^{2t}x(t)$$ and the Fourier transform of $g(t)$ converges then determine the type of sig $x(t)$

$$X(s) = \frac{1}{(s+1)(s+3)}$$



$$G(s) = X(s-2)$$

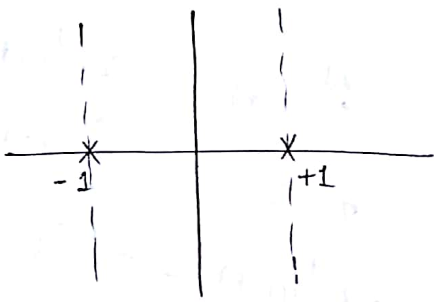
$$G(s) = \frac{1}{(s-2+1)(s-2+3)} = \frac{1}{(s-1)(s+1)}$$

$$= \frac{A}{s-1} + \frac{B}{s+1}$$

$$\frac{1/2}{s-1} + \frac{-1/2}{(s+1)}$$

$0.5\, e^{t} u(t) \quad - \quad 0.5 e^{-t} u(t)$

$\sigma > +1 \qquad \sigma > -1$



$\sigma < +1 \qquad \sigma > -1$

$g(t) = -0.5 e^{t} u(-t) - 0.5 e^{-t} u(t)$

F.T of $g(t)$ convergence means $g(t)$ is stable means ROC of $g(t)$ include $j\omega$ axis means Two sided so $x(t)$ will also be two sided.

Q 4 let $x(t) = 11\,\delta(t) - 7\,\delta(t-3)$. and $x(t)$ is having $\rightarrow X(s)$

if $X(s) = A + B \cdot e^{Cs} \sinh(Ds)$ then the numerical value of const $A, B, C, D = ?$

$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$

Soln $X(s) = 11 - 7\, e^{-3s}$

$= 11 - 7\{$

$e^{\theta} \therefore \sinh\theta + \cosh\theta$

By Sir $11 - 7\, e^{-3s}$

$= 4 + 7 - 7 \cdot e^{-3s}$

$= 4 + 7\left[1 - e^{-3s}\right]$

$= 4 + 7\, e^{-3/2 s}\left[\frac{e^{3/2 s} - e^{-3/2 s}}{2}\right] \cdot 2$

$4 + 14\, e^{-3/2 s} \sinh \frac{3}{2} s$

Q 5 (ESE) A causal sig $x(t)$ has a laplace transform $X(s) = \frac{s}{s^4 - 16}$. if $y(t) = 5x(3t)$ and $Y(s)$ can be expressed in the form $Y(s) = \frac{A \cdot s}{s^4 - a^4}$ determine the values of constants A and a.

$x(t) \longrightarrow \frac{s}{s^4 - 16}$

$y(t) \longrightarrow 5x(3t) \longrightarrow \frac{5}{3} \cdot \frac{s/3}{(s/3)^4 - 16}$

$\frac{5}{3} \cdot \frac{\frac{s}{3}}{\frac{s^4}{3^4} - 16} = \frac{5}{3} \cdot \frac{s \cdot 3^3}{s^4 - 16 \times 3^4}$

$= 5 \times 3^2 \frac{s}{s^4 - 16 \times 3^4}$

$= 45 \frac{s}{s^4 - (6)^4}$

$A = 45$

$a = 6$

(easy) Q6 $x(t) \longrightarrow X(s)$ has partial fraction expansion as given below

$$X(s) = \frac{6}{(s+4)(s+a)} = \frac{2}{(s+4)} + \frac{b}{(s+a)}$$

Value of const a, b ?

Soln

$$\frac{A}{s+4} + \frac{B}{(s+a)}$$

$$\frac{6}{-4+a} = 2 \qquad \frac{6}{-a+4} = b$$

$$6 = -8 + 2a \qquad \frac{6}{-7+4} = b$$

$$14 = 2a \qquad \frac{6}{-3} = b$$

$$\boxed{a = 7} \qquad \boxed{b = -2}$$

(easy) Q7 A causal LTI system has zero initial ~~s~~ conditions and impulse response $h(t)$. its i/p $x(t)$ and o/p $y(t)$ are related through the ~~e~~ linear const coefficient differential eqn

$$\frac{d^2y(t)}{dt^2} + \alpha \frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

Let another sig $g(t)$ be defined as $g(t) = \alpha^2 \cdot \int_0^t h(\tau)d\tau + \frac{dh(t)}{dt} + \alpha h(t)$

where $h(t)$ is the impulse response of the LTI system. if $G(s)$ is Laplace transform of sig $g(t)$ then the no. of poles in $G(s)$ will be

$$Y(s)\left(s^2 + \alpha s + \alpha^2\right) = X(s)$$

$$H(s) = \frac{1}{(s^2+\alpha s+\alpha^2)}$$

$$G(s) = \alpha^2 \cdot \frac{1}{s(s^2+\alpha s+\alpha^2)} + \frac{s^2}{s(s^2+\alpha s+\alpha^2)} + \frac{\alpha s}{s(s^2+\alpha s+\alpha^2)}$$

$$= \frac{(s^2+\alpha s+\alpha^2)}{s(s^2+\alpha s+\alpha^2)} = \frac{1}{s} \text{ Ans}$$

Q. Suppose the following facts are given about sig $x(t)$ with Laplace transform $X(s)$

① $x(t)$ is Real and Even

② $X(s)$ has exactly 4 poles and no zero in the finite s plane.

③ $X(s)$ has a pole at $s = \frac{1}{2} e^{j\pi/4}$

④ $\int_{-\infty}^{+\infty} x(t)\, dt = 4$

the value of $X(s)$ will be:

Soln

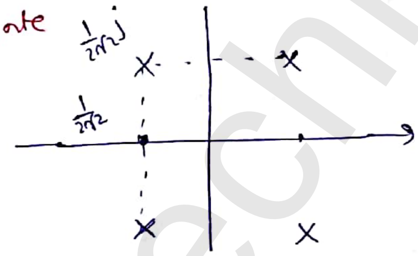
Even signals के अगर pole right side में हैं तो left side में भी होंगे।

$$s = \frac{1}{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)$$

$$= \frac{1}{2}\left(\cos 45 + j\sin 45^\circ\right)$$

$$= \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$$

Real sig's के लिए poles complex conjugate के form में होते हैं



$$\left(s - \frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}\right)$$

$$\left(s - \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}\right)$$

$$\left(s - \frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2$$

$$\left[\left(s - \frac{1}{2\sqrt{2}}\right)^2 + \frac{1}{8}\right]$$

$$\left(s + \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}\right)\left(s + \frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}\right)$$

$$\left(\left(s + \frac{1}{2\sqrt{2}}\right)^2 + \frac{1}{8}\right)$$

$$X(s) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st}\, dt$$

$$X(0) = 4$$

$$X(s) = \frac{K}{\left(\left(s + \frac{1}{2\sqrt{2}}\right)^2 + \frac{1}{8}\right)\left(\left(s - \frac{1}{2\sqrt{2}}\right)^2 + \frac{1}{8}\right)}$$

$$X(0) = 4 = \frac{K}{\frac{2}{8} \times \frac{2}{8}}$$

$$4 = \frac{K\,64}{4}$$

$$K = \frac{16}{64} = 0.25$$

$$= \frac{0.25}{\left(\left(s+\frac{1}{2\sqrt{2}}\right)^2+\frac{1}{8}\right)\left(\left(s-\frac{1}{2\sqrt{2}}\right)^2+\frac{1}{8}\right)}$$

$$= \frac{0.25}{\left(s^2+\frac{1}{8}+\frac{s}{\sqrt{2}}+\frac{1}{8}\right)\left(s^2+\frac{1}{8}-\frac{s}{\sqrt{2}}+\frac{1}{8}\right)}$$

$$= \frac{0.25}{\left(s^2+\frac{s}{\sqrt{2}}+\frac{1}{4}\right)\left(s^2+\frac{1}{4}-\frac{s}{\sqrt{2}}\right)}$$

$$\frac{0.25}{s^4+\cancel{\frac{s^2}{4}}} = \frac{0.25}{\left(s^2+\frac{1}{4}\right)^2-\left(\frac{s}{\sqrt{2}}\right)^2}$$

$$= \frac{0.25}{\left(s^2+\frac{1}{4}\right)^2-\frac{s^2}{2}}$$

$$\boxed{\text{Final Ans} = \frac{0.25}{\left(s^2+\frac{1}{4}\right)^2}}$$

Q9 determine the value of const α, β, y_0, y_0' so that $Y(s) = \frac{s}{(s+1)^2}$

is the laplace transform of differential eq^n $y''(t) + \alpha y'(t) + \beta y(t) = 0$

with $y(0) = y_0$ $y'(0) = y_0'$

$$s^2 Y(s) - s y(0) - y'(0) + \alpha\{sY(s) - y(0)\} + \beta Y(s) = 0$$

$$Y(s)\{s^2 + \alpha s + \beta\} = s y_0 + y_0' + \alpha y_0$$

$$Y(s) = \frac{s y_0 + y_0' + \alpha y_0}{s^2 + \alpha s + \beta} = \frac{s}{s^2 + 2s + 1}$$

~~$y_0 = 1$~~ $\alpha = 2$ $\beta = 1$

$y_0 = 1$ $y_0' = -2$

most important Q in Fourier Tr. (Laplace, Z)

Q.10 A causal LTI-system with impulse response $h(t)$ has the following properties

① when the i/p to the system is $x(t) = e^t$ for all t, the o/p is $y(t) = \frac{11}{12}e^t$ for all t.

② when the i/p to the system is $x(t) = e^{2t}$; for all t, the o/p is $y(t) = \frac{7}{10}e^{2t}$ for all t.

③ The impulse response $h(t)$ satisfies the equation $h(t) = ae^{-2t}u(t) + be^{-2t}u(t)$

the value of const "a" and "b" will be.

in ~~laplace~~ F.T the i/p was complex exponential in case of laplace

the i/p is $x(t) = Ae^{s_0 t}$

$$x(t) = A \cdot e^{s_0 t} \longrightarrow y(t) = A \cdot e^{s_0 t} \cdot H(s)\Big|_{s=s_0}$$

① $e^t \longrightarrow y(t) = e^t \cdot \frac{11}{12}$ $H(1) = \frac{11}{12}$

$$e^{2t} \longrightarrow y(t) = e^{2t} \cdot \frac{7}{10} \text{ — } H(2)$$

$$H(s) = \frac{a}{s+3} + \frac{b}{s+2}$$

$$H(1) = \frac{11}{12} = \frac{a}{4} + \frac{b}{3} \quad \text{— (1)}$$

$$\frac{11}{12} = \frac{3a}{12} + \frac{4b}{12}$$

$$3a + 4b = 11 \quad \text{— (2)}$$

$$\frac{7}{10} = H(2) = \frac{a}{5} + \frac{b}{4}$$

$$\frac{2 \cdot 7}{2 \cdot 10} = \frac{a \cdot 4}{5 \cdot 4} + \frac{5b}{5 \cdot 4}$$

$$14 = a4 + 5b \quad \text{— (2)}$$

$$14 = 4\left(\frac{11 - 4b}{3}\right) + 5b$$

$$42 = 44 - 16b + 15b$$

$$b = 44 - 42$$

$$\boxed{b = 2}$$

$$3a = 11 - 4 \times 2$$

$$11 - 8$$

$$\boxed{a = 1} \text{ Ans}$$

Don't do.

Q11 A Causal LTI system with impulse response $h(t)$ has the following properties

(1) when the i/p to the system is $x(t) = e^{2t}$ for all t, the o/p is $y(t) = \frac{1}{6}e^{2t}$ for all t.

(2) the impulse response $h(t)$ satisfies the differential eq^n $\frac{dh(t)}{dt} + 2h(t) = e^{-4t}u(t) + b\cdot u(t)$ where b is an unknown constt. the value of $H(s)$ will be.

$$e^{2t} \longrightarrow e^{2t}\cdot\frac{1}{6} \quad H(2)$$

$$sH(s) + 2H(s) = \frac{1}{(s+4)} + \frac{b}{s}$$

$$H(s) = \frac{1}{(s+2)(s+4)} + \frac{b}{s(s+2)}$$

$$H(2) = \frac{1}{6} = \frac{1}{4\cdot6} + \frac{b}{2\cdot4}$$

$$\frac{4}{4}\cdot\frac{1}{6} = \frac{1}{4\cdot6} + \frac{b\cdot3}{2\cdot4\cdot3}$$

$$4 = 1 + 3b$$

$$b = 1$$

$$H(s) = \frac{1}{(s+2)(s+4)} + \frac{1}{s(s+2)}$$

$$\frac{1}{s+2}\left\{\frac{1}{(s+4)} + \frac{1}{s}\right\}$$

$$\frac{1}{s+2}\left\{\frac{s+s+4}{s(s+4)}\right\}$$

$$H(s) = \frac{(2s+4)}{(s+2)\,s(s+4)} = \frac{2(s+2)}{(s+2)\,s(s+4)} = \frac{2}{s(s+4)}$$

$$H(s) = \frac{2}{s(s+4)}$$

Q.12 Consider a continuous time LTI system given by II^{nd} order diffⁿ eqⁿ $y''(t) + 3y'(t) + 2y(t) = x(t)$ Here $y(0) = 3$, $y'(0) = 4$ and $x(t) = 4 \cdot e^{-2t} u(t)$ determine.

① zero i/p response of the system.
② zero state response of the system.
③ Total response of the system

Solⁿ

$$s^2 Y(s) - s y(0) - y'(0) + 3 s Y(s) - 3 y(0) + 2 Y(s) = X(s)$$

$$Y(s) \{ s^2 + 3s + 2 \} = X(s) + s y(0) + y'(0) + 3 y(0)$$

$$Y(s) = \frac{X(s)}{s^2 + 3s + 2} + \frac{3s + 4 + 9}{s^2 + 3s + 2}$$

$$Y(s) = \frac{X(s)}{s^2 + 3s + 2} + \frac{3s + 13}{s^2 + 3s + 2}$$

$$X(s) = 4 \frac{1}{(s+2)}$$

$$Y(s) = \frac{4}{(s+2)(s^2+3s+2)} + \frac{(3s+13)}{(s^2+3s+2)}$$

$$y(t) = \frac{A}{(s+2)} + \frac{Bs + C}{(s^2+3s+2)} + \underbrace{\frac{3s+13}{(s+2)(s+1)}}$$

ZIR
(Response when i/p is zero)

$$y_{zi}(t) = \frac{10}{s+1} + \frac{-7}{(s+2)}$$

$$= 10 e^{-t} u(t) - 7 e^{-2t} u(t)$$

$$\frac{4}{(s+2)(s^2+3s+2)} = \frac{4}{(s+2)^2(s+1)}$$

$$= \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

J, at this point Do partial Fraction shortcuts.

First find A by putting $s=-1$ in $\frac{4}{(s+2)^2(s+1)}$

$\boxed{A=4}$

Now $\lim_{s\to\infty} s \cdot F(s)$

$\lim_{s\to\infty} \frac{4 \cdot s}{(s+2)^2(s+1)} = \frac{A \cdot s}{(s+1)} + \frac{B \cdot s}{(s+2)} + \frac{C \cdot s}{(s+2)^2}$

$0 = A + B + 0$

$0 = 4 + B$

$\boxed{B=-4}$

Now put $s=0$

$\frac{4}{4} = 4 - 2 + \frac{C}{4}$

$1 = \cancel{\frac{8}{4}} + 2 + \frac{C}{4}$

$C=4$

$-1 = \frac{C}{4}$

$\boxed{C=-4}$

$$= \frac{4}{s+1} - \frac{4}{(s+2)} - \frac{4}{(s+2)^2}$$

$$\underbrace{y_{zs}(t)}_{\text{zero state response}} = 4e^{-t}u(t) - 4e^{-2t}u(t) - 4t \cdot e^{-2t}u(t)$$

$$y(t) = y_{zi}(t) + y_{zs}(t)$$

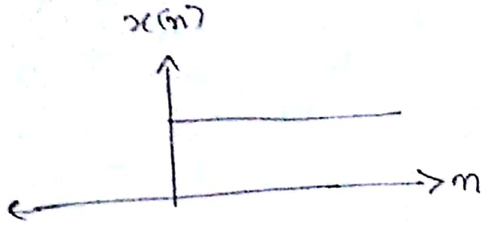
$$y(t) = 14e^{-t}u(t) - 11e^{-2t}u(t) - 4t\,e^{-2t}u(t)$$

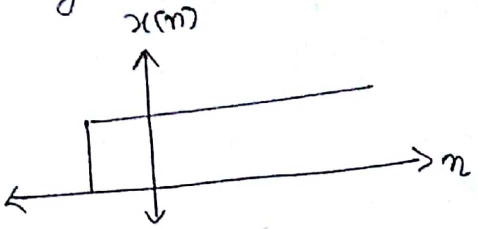
Now verifying our Answer

$y(0)$ should equal to 3

$y(0) = 14 - 11 \cancel{-4} = 3$

X(z) को देख कर signal की causality पता करने का तरीका :-





Right sided
+
Causal signal

bcz $x(n) = 0 \; ; \; n < 0$

Right sided
+
Non causal sig bcz

$x[n] \neq 0 \; ; \; n < 0$

by seeing, in time domain we can tell about causality directly.

But How to tell by seeing the z transform of the signal.

$$x(n) = u(n) \longleftrightarrow \frac{1}{1-z^{-1}} = \frac{z}{z-1} \; ; \; |z| > 1$$

in z transform no. of zeros are equal to no. of poles. in above $\frac{z}{z-1}$ we have one pole and one zero

$$\frac{z \leftarrow \text{zero}}{z-1 \leftarrow \text{pole}}$$

$$x(n) = u(n+1) \longrightarrow \frac{z}{1-z^{-1}} = \frac{z^2}{z-1} \; ; \; |z| > 1$$

z^2 ← two zeros

$z-1$ ← one pole.

It means another pole is at ∞

so RoC can't include pole. so

$\underbrace{1 < |z| < \infty}$

this implies non causality

$$x(n) = u(-n) \longleftrightarrow \frac{1}{1-z}$$

1 ← no zero ← this zero will be present at ∞

$1-z$ ← one pole.

$|z| < 1$

$$x(n) = u[-n+1] \longleftrightarrow \frac{1}{z(1-z)}$$

1 ← two zeros will be present at ∞

z → one pole at origin.

$0 < |z| < 1$

↑

so ~~|z|~~ RoC don't include ~~origin~~ pole at origin

Z Transform [not unique so ROC defined] [11 question]

$a^n u(n) \xleftrightarrow{z} \dfrac{1}{1-az^{-1}}$; $|z|>|a|$

$-a^n u(-n-1) \xleftrightarrow{z} \dfrac{1}{1-az^{-1}}$; $|z|<|a|$

① For a right sided sig to be causal, its ROC should be outside the outermost pole including $z=\infty$.

② For a left sided sig to be anticausal, its ROC should be inner to the innermost pole including $z=0$

Z tf $\Rightarrow$ no. of zero = no. of pole if something is less then it will be present at ∞.

$x(n) = u(n) \xleftrightarrow{z} \dfrac{1}{1-z^{-1}} = \dfrac{z}{z-1}$; $|z|>1$

$x(n) = u(n+1) \xleftrightarrow{z} \dfrac{z}{1-z^{-1}} = \dfrac{z^2}{z-1}$; $1<|z|<\infty$

$\to$ 2 zero

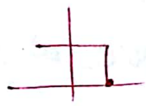
$\to$ 1 pole, another pole at ∞

$x(n) = u(-n) \longleftrightarrow \dfrac{1}{1-z}$; $|z|<1$

no zero

pole

$x(n) = u[-n+1] \longrightarrow \dfrac{z}{1-z^{-1}}$



$\longrightarrow \dfrac{z^{-1}}{1-z} = \dfrac{1}{z(1-z)}$; $0<|z|<1$

$x(n) = 0$; $n<0$

cond^n for causal

$x(n) = 0$; $n>0$

cond for Anti-Causal

Q1 Consider a right sided sig $x(n)$ with $X(z) = \frac{z^3}{z-1}$

① Is the signal causal

② determine $x[n]$

Ans - $\frac{z^3}{z-1}$ ← 3 zeros : $1 < |z| < \infty$

↖ 1 pole   ROC don't include ∞ so non causal

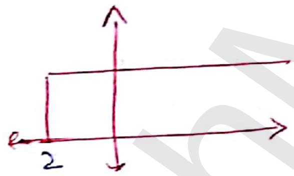
↘ other 2 poles are at ∞. so ROC will not include ∞.

$X(z) = z^2 \cdot \frac{z}{z-1}$

$\Downarrow$

$u(n)$

$x(n) = u[n+2]$



Right sided + Non causal

Q 2 Suppose we are given 5 facts about a particular LTI system with impulse response $h[n]$ and transfer func $H(z)$

① $h(n)$ is real

② $h(n)$ is right sided

③ $\lim_{z \to \infty} H(z) = 1$

④ $H(z)$ has exactly two zeros.

⑤ $H(z)$ has one of its pole at non real location on the circle defined by $|z| = \frac{3}{4}$

Is the system causal

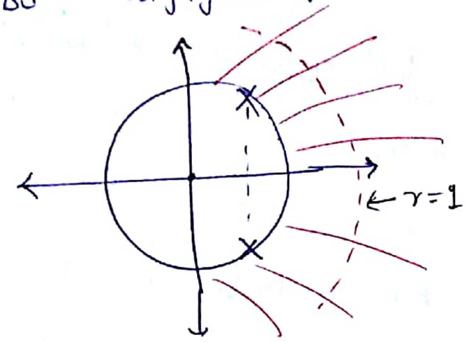
Is the system stable.

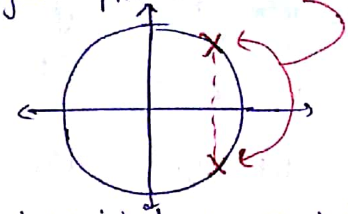
Soln b/c ③ $\lim_{z \to \infty} H(z) = 1$ mean at $z = \infty$ no pole and no zeros so we have two zeros and two poles.

we have exactly two zeros means we must have exactly two poles else the extra pole will lie at ∞.

& $|z| > \frac{3}{4}$. these two things ⇒ that system is causal bcz $h(n) =$ ~~real~~ is real $\Rightarrow$ poles will be conjugate symmetric i.e

so conjugate pole.





$h(n)$ is right sided $\Rightarrow |z| > \frac{3}{4}$

stable. $|z| > \frac{3}{4} \Rightarrow$ $|z|$ include unit circle so stable.

Q2 let $x(n) = 2^n u(n)$. $X(z)$ for $z = 3e^{j\omega}$ can be thought of as the DTFT of

Q3 let $x(n)$ sig is having z transform $X(z)$ and $X(z)$ on the circle $z = 2e^{j\omega}$ is given by $X(2e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$. The value of sig $x(n)$ will be.

$$X(2e^{j\omega}) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

$$= \frac{1}{1 - \frac{2}{3} \cdot \frac{1}{2}e^{-j\omega}} = \frac{1}{1 - \frac{2}{3}(2e^{j\omega})^{-1}}$$

$$= \frac{1}{1 - \frac{2}{3}z^{-1}}$$

$$x(n) = \left(\frac{2}{3}\right)^n u(n)$$

//

brilliant

Q4 Let $x(n)$ be a sig whose rational z transform contains a pole at $z = \frac{1}{2}$. it is given that $x_1(n) = \left(\frac{1}{4}\right)^n x(n)$ is absolutely summable and $x_2(n) = \left(\frac{1}{8}\right)^n x(n)$ is not absolutely summable. determine the type of sig $x(n)$

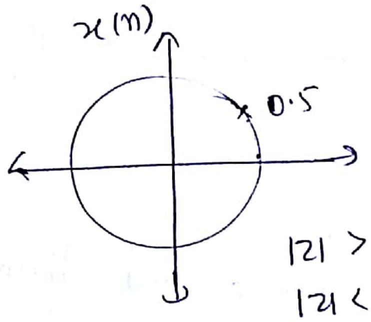
$x_1(n)$ absolutely summable means its ROC include unit circle

$x_2(n)$ not " " " " " don't include unit-circle.

$x(n) \longrightarrow X(z)$

Scaling in z domain property

$$z_0^n \cdot x(n) \overset{z}{\longleftrightarrow} X\left(\frac{z}{z_0}\right)$$



$\cancel{x_1(n)}$

$|z| > \left|\frac{1}{4}\right|$

$\frac{1}{4}$

$|z| > 0.5$

$|z| < 0.5$

$x_1(n) =$

$|z| > 0.5 + 0.25 \quad |z| > 0.75$

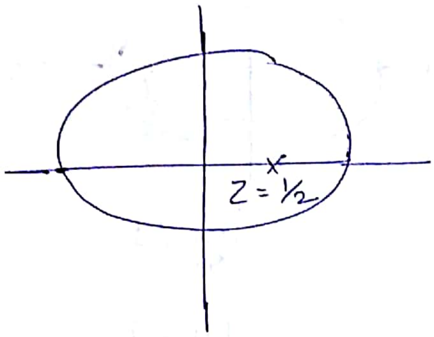
$|z| < 0.5 + 0.25 \quad |z| < 0.75$

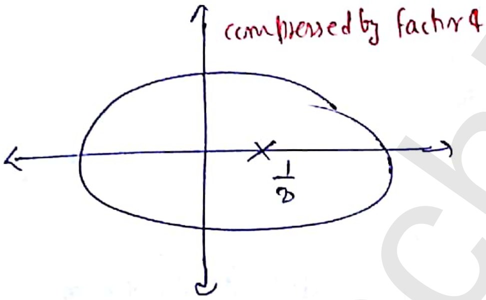
$x(n) \longrightarrow X(z)$

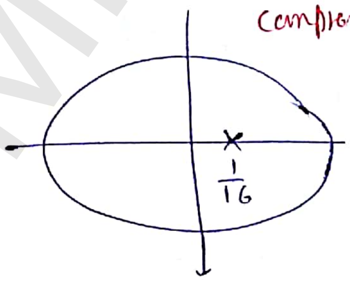
$x_1(n) = \left(\frac{1}{4}\right)^n u(n) \xleftrightarrow{Z} X(4z)$ → Pole of $X(z)$ will be compressed by factor of 4.

$x_2(n) = \left(\frac{1}{8}\right)^n u(n) \longleftrightarrow X(8z)$

$z_0^n \cdot x(n) \to X\left(\frac{z}{z_0}\right)$







① If $x(n)$ is left sided. i.e $|z| < \frac{1}{2}$

$x_1(n)$ and $x_2(n)$ both are unstable i.e not absolutely summable
$|z| < \frac{1}{8}$ $\quad |z| < \frac{1}{16}$
but we are given than $x_1(n)$ is summable.

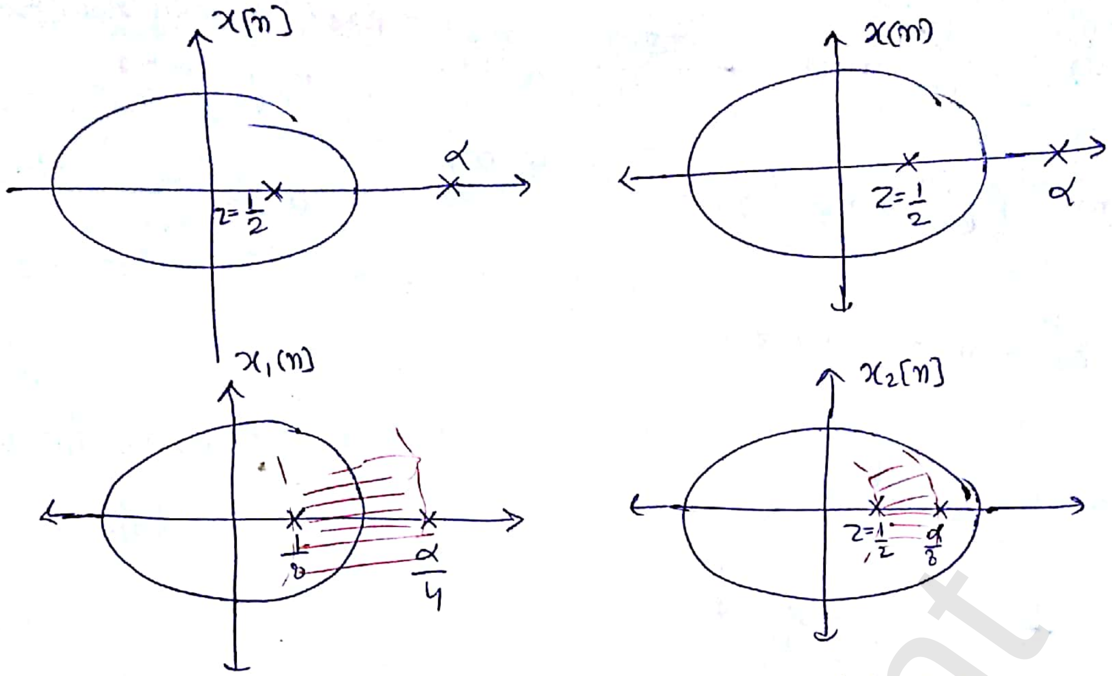
② If $x(n)$ is Right sided sig ie $|z| > \frac{1}{2}$

$x_1(n)$ and $x_2(n)$ are absolutely summable. but $x_2(n)$ is not summable.
$|z| > \frac{1}{8}$ $\quad |z| > \frac{1}{16}$

③ $x(n)$ can't be a finite sig bcz then ROC will be entire z plane, it means it will include $z = \frac{1}{2}$ pole and we know ROC don't include pole.

So $x(n)$ is two sided sig. it means there exist one more pole. let that pole exist at α.

तो हमें α की ऐसी value choose करनी है जिसके लिए $x_1(n)$ stable हो and $x_2(n)$ unstable.

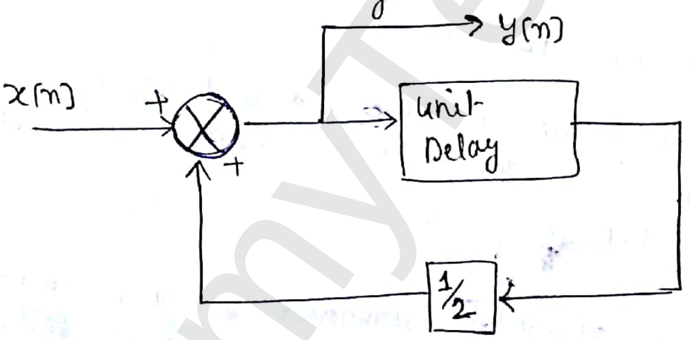


$\left|\frac{\alpha}{4}\right| > 1$ $\qquad$ $\left|\frac{\alpha}{8}\right| \leq 1$

$|\alpha| > 4$ $\qquad$ $|\alpha| \leq 8$

$4 < |\alpha| \leq 8$

Q5
Consider a descrete time LTI system as shown below find h(n)



There is f/b so IIR filter, if no f/b so FIR filter means.

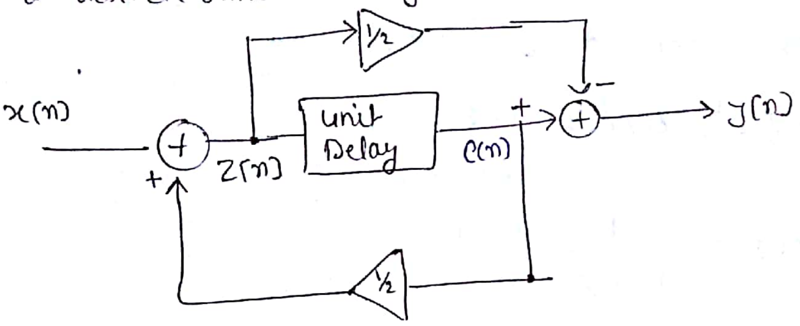
$y(n) = x(n) + \frac{1}{2} y[n-1]$

$y(n) - \frac{1}{2} y(n-1) = x(n)$

$Y(z) - \frac{1}{2} Y(z) z^{-1} = X(z)$

$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2} z^{-1}} = \left(\frac{1}{2}\right)^n u[n] = h[n]$

Q6 Consider a descrete time LTI system as shown below find h(n)



Soln

$Z(n) = C(n) +$

~~Z = C + x~~

$Z = x + \frac{C}{2}$

$Z(n) = x(n) + \frac{C(n)}{2}$

$C(n) = x(n-1) + \frac{C(n-1)}{2}$

$C(n) - \frac{C(n-1)}{2} = x(n-1)$

$y(n) = C(n) - \frac{1Z(n)}{2}$

$y(n) = C(n) - \frac{1}{2}\left[x(n) + \frac{1}{2}C(n)\right]$

$y(n) = C(n) - \frac{1}{2}x(n) - \frac{1}{4}C(n)$

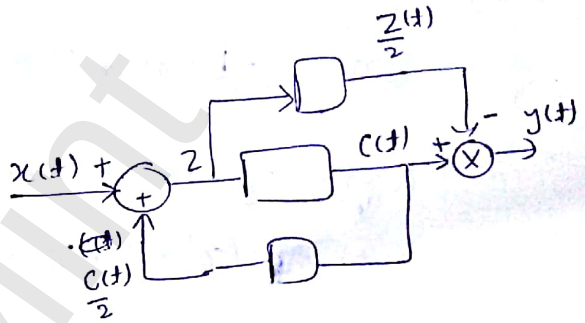
$y(n) + \frac{1}{2}x(n) = C(n)\left[1 - \frac{1}{4}\right]$

$Y(z) + \frac{1}{2}X(z) = C(z)\,\frac{3}{4}$

$C(z) - \frac{1}{2}C(z)\cdot z^{-1} = X(z)\cdot z^{-1}$

$C(z)\left\{1 - \frac{1}{2}z^{-1}\right\} = X(z)\cdot z^{-1}$

$C(z) = X(z)\cdot\frac{z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)}$



$y(t) = C(t) - \frac{Z}{2}(t)$

$Z(t)$

$Z(t) = x(t) + \frac{1}{2}C(t)$

$Z(t-1)$

$Y(z) + \frac{1}{2}X(z) = \frac{3}{4}\,\frac{X(z)}{\frac{z}{2}(2z-1)}$

$Y(z) = X(z)\left\{\frac{3}{(2z-1)} - \frac{1}{2}\right\}$

$\frac{Y(z)}{X(z)} = \left\{\frac{6 - 2z + 1}{2(2z-1)}\right\}$

$\frac{Y(z)}{X(z)} = \frac{7 - 2z}{2(2z-1)}$

By Sir

$$z[n] = x(n) + \frac{1}{2} z[n-1]$$

$$\left(1 - \frac{1}{2} z^{-1}\right) Z(z) = X(z)$$

$$Z(z) = \frac{1}{\left(1 - \frac{1}{2} z^{-1}\right)} \cdot X(z)$$

$$y(n) = -\frac{1}{2} z[n] + z[n-1]$$

$$Y(z) = -\frac{1}{2} Z(z) + Z(z) \cdot z^{-1}$$

$$Y(z) = \left(z^{-1} - \frac{1}{2}\right) Z(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\left(z^{-1} - \frac{1}{2}\right)}{\left(1 - \frac{1}{2} z^{-1}\right)} \qquad \frac{\left(\frac{1}{z} - \frac{1}{2}\right)}{\left(1 - \frac{1}{2z}\right)} = \frac{\frac{2-z}{2z}}{\frac{2z-1}{2z}}$$

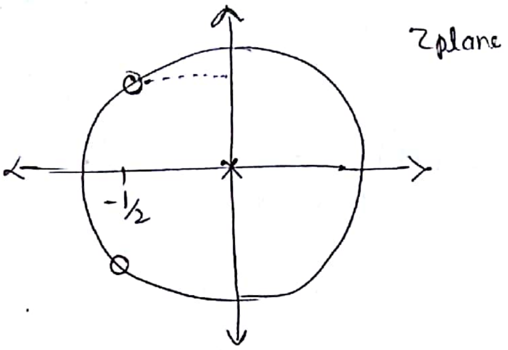
$$= \frac{z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)} - \frac{1/2}{\left(1 - \frac{1}{2} z^{-1}\right)}$$

$$h(n) = \left(\frac{1}{2}\right)^{n-1} u(n-1) - \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{n} u(n)$$

$$= -\left(\frac{1}{2}\right)^{n+1} u(n)$$

Q7 The pole zero plot for a discrete time system is shown below

F.T, Lap, Z trf
~~This is the~~



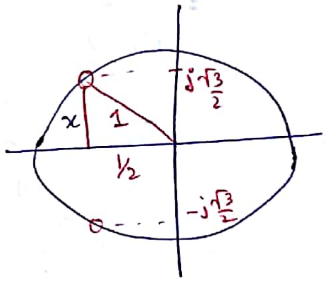
Here the circle has radius 1. it is known that when the i/p is 1 for all n, the o/p is also 1 for all n. determine impulse response of the system.

$$x(n) = 1 \longrightarrow y(n) = 1$$

in case of z t/f

$$z_0^n \longrightarrow y_0(n) = z_0^n \cdot H(z)\Big|_{z=z_0}$$

$$= \frac{K\left(z+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(z+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}{z}$$



$$H(z) = \frac{K\left\{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}\right\}}{z}$$

$$x = \sqrt{1^2-\left(\tfrac{1}{2}\right)^2}$$

$$x = \sqrt{1-\tfrac{1}{4}}$$

$$x = \frac{\sqrt{3}}{2}$$

$$\frac{K\left(z^2+\frac{1}{4}+z+\frac{3}{4}\right)}{z}$$

$$H(z) = \frac{K(z^2+z+1)}{z}$$

$$x(n) = 1^n \longrightarrow y(n) = (1)^n \cdot \underset{\sim}{1}$$

$$\hookrightarrow H(z)\Big|_{z=1} = 1$$

$$H(z)\Big|_{z=1} = 1 = K\frac{(1+1+1)}{1}$$

$$\boxed{K = \frac{1}{3}}$$

$$= \frac{1}{3}\frac{[z^2+z+1]}{z}$$

$$H(z) = \frac{1}{3}[z+1+z^{-1}]$$

$$h(n) = \frac{1}{3}[\delta(n+1) + \delta(n) + \delta(n-1)]$$

Q 8 Let $X(z) = \frac{1}{(1-2z^{-1})^2}$ with ROC $|z|>2$

determine the values of $x(2)$ and $x(9)$

$$u(n) = \frac{z}{z-1}$$

$$X(z) = \frac{1}{(1-\frac{2}{z})^2}$$

$$(a)^n n u(n) = \frac{za}{(z-a)^2}$$

$$= \frac{z^2}{(z-2)^2}$$

$$(a)^n . n u(n) = \frac{za}{(z-a)^2}$$

$$X(z) = \frac{z}{2} \frac{z.2}{(z-2)^2}$$

$$\Downarrow$$

$$= (2)^n . n u(n)$$

$$\frac{(2)^{n+1}}{2}(n+1)\, u(n+1)$$

$$x(2) = \frac{(2)^3}{2} \times 3 = \frac{8\times 3}{2} = 12$$

$$x(9) = \frac{2^{10}}{2} \times 10 = 2^9 \times 10 = 5120$$

same qu^n ROC < |2| $\checkmark$ Dont

$$-(2)^n n\, u[-n-1]$$

$$-(2)^{n+1} (n+1)\, u[-(n+1)-1]$$

$$-(2)^{n+1} (n+1)\, u[-n-1-1]$$

$$(-2)^{n+1} (n+1)\, u[-n-2]$$

$$x(2) = (-2)^3 (3) \cdot u[-2-2]$$ $\boxed{x(2) = \frac{1}{4}}$ Ans.

$$= \frac{1}{2}$$

$$X(z) = \frac{z}{2} \cdot \frac{2z}{(z-2)^2} \quad ; \quad |z| < 2$$

Q9 Consider a descrete time LTI system governed by a difference equation $y(n) = \frac{1}{4}y(n-1) + \frac{3}{8}y[n-2] + 5x(n) + 10x[n-1]$.
if the system is excited by a ~~unique~~ unit step function then the value of $y[\infty]$ will be = ?

Soln

$$Y(z) = \frac{1}{4}Y(z)\cdot z^{-1} + \frac{3}{8}Y(z)\cdot z^{-2} + 5X(z) + 10X(z)\cdot z^{-1}$$

$$Y(z)\left\{1 - \frac{z^{-1}}{4} - \frac{3}{8}z^{-2}\right\} = X(z)\left\{5 + 10z^{-1}\right\}$$

$$Y(z) = \frac{z}{z-1}\frac{(5+10z^{-1})}{(1 - \frac{z^{-1}}{4} - \frac{3}{8}z^{-2})}$$

$$Y(z) = \frac{z}{z-1}\frac{(5+\frac{10}{z})}{\left[1 - \frac{1}{4z} - \frac{3}{8z^2}\right]}$$

$$Y(z) = \frac{\cancel{z}}{(z-1)\cancel{z}}\frac{(5z+10)}{(8z^2-2z-3)}8z^2$$

$$Y(z) = \frac{5(z+2)8z^2}{(z-1)(8z^2-2z-3)}$$

$$\lim_{z\to 1}(1-z^{-1})\frac{5(z+2)8z^2}{(z-1)(8z^2-2z-3)}$$

$$\lim_{z\to 1}\frac{5(z+2)8z}{\cancel{z-1}(8z^2-2z-3)}$$

$$\frac{5(3)8}{8-2-3} = \frac{40\times 3}{8-5} = 40$$

method (2) $y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y[n-2] = 5u(n) + 10u(n-1)$

$$y(\infty) - \frac{1}{4}y[\infty] - \frac{3}{8}y(\infty] = 5 \cdot u(\infty) + 10u[\infty]$$

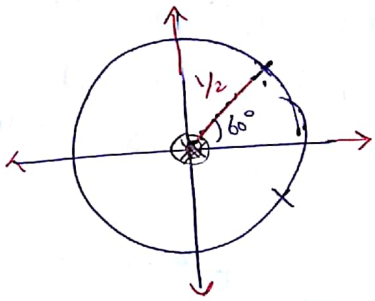
$$y[\infty]\left(1-\frac{1}{4}-\frac{3}{8}\right) = 15$$

$$y(\infty) = \frac{15}{\left(1-\frac{1}{4}-\frac{3}{8}\right)} = \frac{15 \cdot 8}{(8-2-3)} = \frac{15\times 8}{3} = 40$$

Q 10 We are given the following 5 facts about a descrete time sig $x(n)$ with Z transform $X(z)$

1. $x(n)$ is real and causal.
2. $X(z)$ has exactly 2 poles.
3. $X(z)$ has two zeros at the origin.
4. $X(z)$ has a pole at $z = \frac{1}{2}e^{j\pi/3}$
5. $X(1) = \frac{8}{3}$

determine $X(z)$



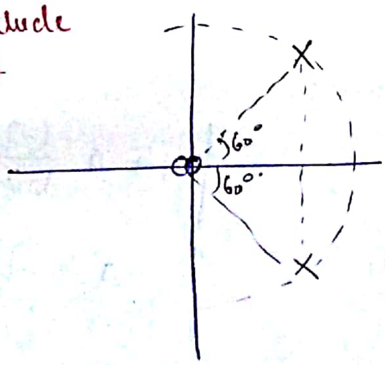
** For causal no. of zero shld equal or less than no. of pole

than pole will been be at ∞ so bcz if no. of zero > than pole

so our system will not be then causal so.

$x(n)$ is given causal it means no. of zeros less or equal to poles.

means ROC can include zeros but ROC don't include poles.



$$X(z) = \frac{kz^2}{(z - \frac{1}{2}e^{j\pi/3})(z - \frac{1}{2}e^{-j\pi/3})}$$

$$= \frac{kz^2}{z^2 - \frac{z}{2} + \frac{1}{4}} = \frac{2z^2}{z^2 - \frac{z}{2} + \frac{1}{4}}$$

$$\frac{k}{1 - \frac{1}{2} + \frac{1}{4}} = \frac{8}{3}$$

$$\boxed{k = 2}$$

Q11 The following is known about a discrete time LTI system with i/p $x(n)$ and o/p $y(n)$

① if $x(n) = (-2)^n$ for all n ; $y(n) = 0$ for all n

② if $x(n) = \left(\frac{1}{2}\right)^n u(n)$ then $y(n) = \delta[n] + a\left(\frac{1}{4}\right)^n u[n]$

where "a" is a constant.

The value of constt "a" will be

$$\cancel{x(n)} = (z_0)^n \longrightarrow (z_0)^n \cdot H(z_0)$$

①

$$(-2)^n \xrightarrow[H(z)]{h[n]} (-2)^n \cdot H(-2)$$

$$(-2)^n \cdot 0 \qquad \underline{H(-2) = 0}$$

$$(-2)^n \xrightarrow[H(z)]{h[n]} 0$$

$$X(z) = \frac{z}{z - 1/2}$$

$$Y(z) = 1 + a\frac{z}{z - 1/4}$$

$$H(z) = \frac{1 + a\frac{z}{z - 1/4}}{\frac{z}{z - 1/2}} = 0$$

$$1 + a\frac{(-2)}{(-2) - 1/4} = 0$$

$$\frac{a(-2)}{-2 - \frac{1}{4}} = -1$$

$$a(-2) = \frac{9}{4}$$

$$\boxed{a = -\frac{9}{8}}$$

DFT

why do we use DFT →

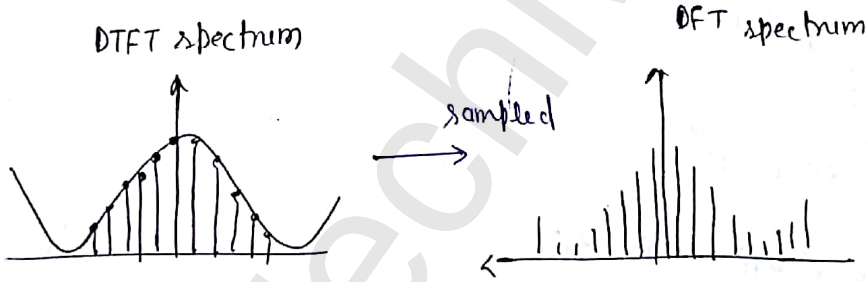
CTFT and DTFT can't ~~respresent~~ do Digital Slg processing.

why DFT → spectrum is continuous in nature.

in DFT → frq spectrum is Discrete in nature.

Since in Both CTFT and DTFT, the spectrum is continuous in nature so an ∞ amount of memory is required to store the spectrum which is practically impossible.

so in DFT, the spectrum of DTFT is sampled to convert it into a discrete spectrum. since sampling in one domain is always periodicity in other domain. so in DFT, the slg both in time as well as in frq is discrete as well as periodic



Remb^r

$$X[k] = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}k \cdot n}$$

Remb^r.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j\frac{2\pi}{N} \cdot n \cdot k}$$

For non-periodic generally.

①	$x(t)$	$\longrightarrow$	$X(s)$ LT	NP
②	$x(n)$	$\longrightarrow$	$X[z]$ ZT	NP
③	$x(t)$	$\longrightarrow$	$X(\omega)$ CTFT	NP
④	$x[n]$	$\longrightarrow$	$X(e^{j\omega})$ DTFT	P

Continuous.

? Doubt. How Slg becomes periodic in time domain when we sample it in Frq domain.

$$\boxed{e^{-j\pi k} = (-1)^k} \boxed{= e^{j\pi k}}$$

Q Consider a discrete time sig $x[n] = \{1, 2, 1, 0\}$; $N = 4$
determine $x[k]$

Sol^n. $N = 4$

$x(n) = \{1 \; 2 \; 1 \; 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N} \cdot k \cdot n}$$

$$X[1] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{4} \cdot 1 \cdot n} =$$

$$X(1) = x[0] \cdot + x[1] \cdot e^{-j\frac{\pi}{2}} + x[2] \cdot e^{-j\pi} + x[3] \cdot e^{-j\frac{\pi}{2} \cdot 3}$$

$$= 1 + 2(-j) + 1(-1) + 0.$$

$$= 1 - 2j + 1$$

$$= -2j$$

$$X[0] = \sum_{n=0}^{N-1} x(n) \cdot e^{0}$$

$$= x[0] + x[1] + x[2] + x[3]$$

$$= 1 + 2 + 1 + 0$$

$$= 4$$

$$X[2] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{4} \cdot 2 \cdot n}$$

$$= x[0] \cdot + x[1] \cdot e^{-j\pi} + x[2] \cdot e^{-j2\pi} + x[3] e^{-j3\pi}$$

$$= 1 + 2 \cdot (-1) + 1(1) + 0$$

$$= 1 - 2 + 1$$

$$= 0$$

$$X[3] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{4} \cdot 3 \cdot n}$$

$$= x[0] + x[1] \cdot e^{-j\frac{3\pi}{2}} + x[2] e^{-j3\pi} + x[3] \cdot e^{-j3\pi \cdot 3}$$

$$= 1 + 2(-j) + 1 \cdot (-1) + 0$$

$$= 1 - 2j - 1$$

$$= -2j$$

The drawback of this method is that it consumes time and memory.

$$X[K] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{matrix} 1+2+1+0 = 4 \\ 1-2j-1+0 = -2j \\ 1-2+1+0 = 0 \\ 1+2j-1+0 = +2j \end{matrix}$$

1st Row and 1st Column → all 1

3rd Row and 3rd " → 1 −1 +1 −1

2 R and 2nd " → (−j)

We know that $x[n]$ and $X[K]$ are periodic

$x(n) = \{ \ldots\ldots 1, 2, 1, 0, \underset{\uparrow}{1}, 2, 1, 0, 1, 2, 1, 0, \ldots\ldots \}$

$X[K] = \{ \ldots\ldots 4, -2j, 0, 2j, \underset{\uparrow}{4}, -2j, 0, 2j, 4, -2j, 0, 2j \ldots \}$

① Time shifting property

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{matrix} = 4 \\ = 1-1+2j = 2j \\ = 1+1-2 = 0 \\ = 1+0-1-2j = -2j \end{matrix}$$

$x(n) \xrightarrow{F.T} X[k]$

$x(n-n_0) \xrightarrow{FT} e^{-j\frac{2\pi}{N}k \cdot n_0} X[K]$

$x[n+n_0] \xrightarrow{FT} e^{j\frac{2\pi}{N} \cdot k \cdot n_0} X[K]$

$x(n) = 1 \ 2 \ 1 \ 0$

$x(n-1) = \{0 \ 1 \ 2 \ 0\}$

$x(n-2) = \{1 \ 0 \ 1 \ 2\} \xrightarrow{FT} e^{-j\frac{2\pi}{4} \cdot k \cdot 2} X[K] = e^{-j\pi k} \cdot X(K)$

$= (-1)^K X(K) = \{4, 2j, 0, -2j\}$

② Shifting in Freq^n domain

$$x(n) \xrightarrow{FT} X(k)$$

$$e^{j\frac{2\pi}{N}\cdot k_0 \cdot n} x(n) \xrightarrow{FT} X(k-k_0)$$

$$e^{-j\frac{2\pi}{N}k_0 \cdot n} x(n) \longrightarrow X(k+k_0)$$

$$x(k) = \{\cdots 4 \quad -2j \quad 0 \quad 2j \cdots\} \; 4, -2j, 0, 2j \ldots\}$$

$$x(k+2) = \{0 \quad 2j \quad 4 \quad -2j\}$$

$$e^{-j\frac{2\pi}{4}\cdot 2\cdot n} x(n) \xrightarrow{FT} X(k+2)$$

$$e^{-j\pi n} x(n) \longrightarrow (-1)^n \cdot x(n) = \{1 \; -2 \; 1 \; 0\}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} = \begin{matrix} 1-2+1+0 = 0 \\ = 1+2j+\cancel{0}1\cdot+0 = 2j \\ = 1+2+1+0 = 4 \\ = \cancel{0+0} \; 1-2j-1+0 = -2j \end{matrix}$$

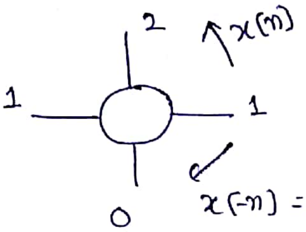
③ Time reversal property

$$x(n) \longrightarrow X[K]$$

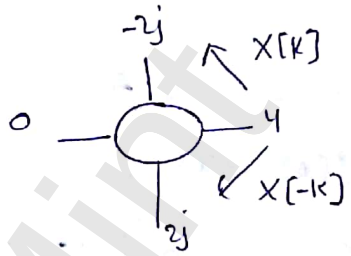
$$x[-n] \longrightarrow X[-K]$$

$$x[n] \longrightarrow$$

$$x(n) = \{1\ 2\ 1\ 0\} \xrightarrow{FT} X(K) = \{4\ -2j\ 0\ 2j\}$$



$x(-n) = \{1, 0, 1, 2\}$



$$X[-K] = \{4\ \ 2j\ \ 0\ \ -2j\}$$

$$x[-n] \xleftrightarrow{FT} X[-K]$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$x[N-n] \xleftrightarrow{FT} X[N-K]$$

N point DFT – ~~Here~~ its 4 point DFT {this N can't be any thing it will only 4 in this case}

Proof for this

$x[N-n]$

$$y(n) = x[4-n] = x[-n] = \{1\ 0\ 1\ 2\}$$

$$y[0] = x[4] = 1$$

$$y[1] = x[3] = 0$$

$$y[2] = x[2] = 1$$

$$y[3] = x[1] = 2$$

④ Conjugation symmetry property

$$x[n] \longleftrightarrow X[K]$$

$$x[-n] \longrightarrow X[-K]$$

$$x^*[n] \xleftrightarrow{FT} X^*[-K]$$

$$= X^*[N-K]$$

now case ①

when $x(n)$ is real ⇒

$$x^*(n) = x[n]$$

Rem^br on Tips.

$$\boxed{X^*[-K] = X^*[N-K] = X[K]}$$

very important

Q.1 Let $X[K]$ with K from 0 to 13 be a 14 point DFT of real sequence $x[n]$. The first 8 samples are given by ~~x~~.

$X[0] = 12$

$X[1] = -1+j3$

$X[2] = 3+j4$

$X[3] = 1-j5$

$X[4] = -2+j2$

$X[5] = 6+j3$

$X[6] = -2-j3$

$X[7] = 10$

remaining values of $X[K]$ will be

Sol^n

$$X[K] = X^*[N-K]$$

$X[8] = X^*[14-8] = X^*[6] = -2+j3$

$X[9] = X^*[5] = 6-j3$

$X[10] = X^*[4] = -2-j2$

$X[11] = X^*[3] = 1+j5$

$X[12] = X^*[2] = 3-j4$

$X[13] = X^*[1] = -1-j3$

⑤ Convolution in time domain :

$$x_1[n] \xrightarrow{FT} X_1[k]$$

$$x_2[n] \xrightarrow{FT} X_2[k]$$

$$x_1[n] \circledast x_2[n]$$

symbol for circular convolution

$$x(n) = \{ \ldots \quad 1 2 1 0 \; 1 2 1 0 \; 1 2 1 0 \ldots \}$$

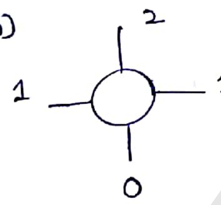
$$X[k] = \{ \ldots \quad 4 \; -2j \; 0 \; 2j \ldots \}$$

$$x[n] \circledast x[n] = X[k] \cdot X[k] = \{4 \; -2j \; 0 \; 2j\}\{4 \; -2j \; 0 \; 2j\}$$

$$= \{16 \; -4 \; 0 \; -4\}$$

$$\{1, 2, 1, 0\} \circledast \{1 \; 2 \; 1 \; 0\}$$

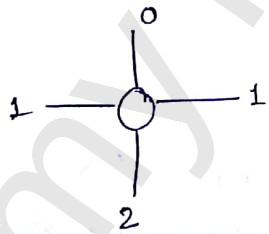
Fix $x(n)$



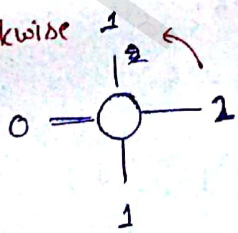
$$y(0) = 1\times1 + 0\times2 + 1\times1 + 2\times0 = 1+1 = 2$$

y

Reversed $x(n)$



anticlockwise shift
$x(n+1)$

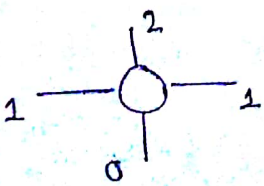


$$y(1) = 2\times1 + 1\times2 + 0\times1 + 1\times0$$

$$= 2+2 = 4$$

y[2]

$x[-n+2]$



$$y[2] = 1 + 4 + 1 + 0 = 6$$

$$y[3] = 0 + 2 + 2 + 0 = 4$$

$\{2, 4, 6, 4\}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ 0 \\ -4 \end{bmatrix}$$

← the same we calculated by $X[k] \cdot X[k]$

⑥
Multiplication in time

$x_1[n] \xrightarrow{FT} X_1[k]$

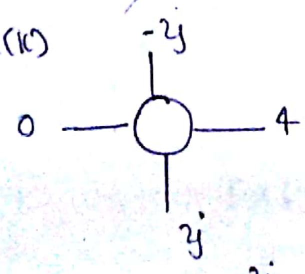
$x_2[n] \longrightarrow X_2[k]$

$x_1[n] \cdot x_2[n] \longrightarrow \frac{1}{N}\left[X_1[k] \circledast X_2[k]\right]$

$\frac{1}{N}$ → bcz sig periodic in N

$X_1[k] = \{4 \quad -2j \quad 0 \quad 2j\}$

X(K)



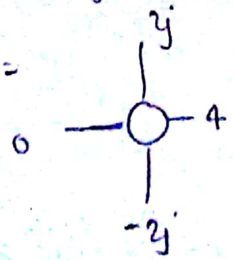
$y[0] = 16 + 4 + 0 + 4 = 24$

$y[1] = -8j - 8j + 0\ 0 = -16j$

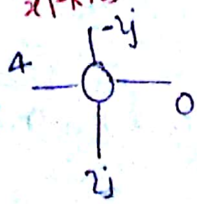
$y[2] = 0 \pm 4 + 0 - 4 = -8$

$y[3] = 8j + 0 + 0 + 8j = 16j$

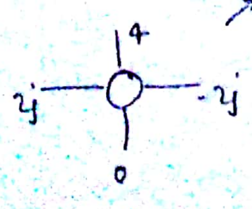
$X_1[-k]$ =

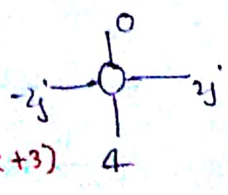


$x[-k+2]$



$x[-k+1]$





$x[-k+3]$

$$\frac{1}{N}\{X(k) \circledast X(k)\}$$

24
-16.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 24 \\ -16j \\ -8 \\ 16j \end{bmatrix} = \begin{bmatrix} 16 \\ \cancel{+16j+8j} \\ 24-16+8-16 \\ \\ \end{bmatrix}$$

Ans will be $\{\ 6 \quad -4j \quad -2 \quad 4j\ \}$

~~Time expansion property~~

~~$x[n] \xrightarrow{FT} X[K]$~~

~~$x[\frac{n}{m}] \xrightarrow{FT}$~~

Gate EC

Q. Consider a sig $x[n] = \{2, 3, 2, 1\}$ with fundamental period 4 having the Fourier transform $X[K] = \{8 \ -2j \ 0 \ 2j\}$

another sig $y[n] = \{2, 0, 0 \ 3 \ 0 \ 0 \ 2 \ 0 \ 0 \ 1 \ 0 \ 0\}$

then determine $\left|\frac{Y[5]}{Y[8]}\right|$

$y[n] = x[\frac{n}{3}] = \{\underset{\uparrow}{2}, 0 \ 0 \ 3 \ 0 \ 0 \ 2 \ 0 \ 0 \ 1 \ 0 \ 0\}$

$Y[K] = \{X[K] \ X[K] \ X[K]\}$

$= \{\underset{\uparrow}{8} \ -2j \ 0 \ 2j \ 8 \ -2j \ 0 \ 2j \ 8 \ -2j \ 0 \ 2j\}$

$\left|\frac{Y[5]}{Y[8]}\right| = \left|\frac{-2j}{8}\right| = \frac{1}{4}$

(7) Time expansion :-

$x[n] \xrightarrow{FT} X[K]$

$x[\frac{n}{m}] \xleftrightarrow{FT} \{\underbrace{X[K], X[K], X[K]}_{m \text{ times}}\}$

$x[\frac{n}{3}] \xleftrightarrow{FT} \{X[K] \ X[K] \ X[K]\}$

⑧ Central ordinate Theorem

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N} \cdot k \cdot n}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{j\frac{2\pi}{N} \cdot n \cdot k}$$

$$X[0] = \sum_{n=0}^{N-1} x[n]$$

$$x(0) = \frac{1}{N} \sum_{k=0}^{N-1} X[k]$$

Proof-

$$x(n) = \{1 \quad 2 \quad 1 \quad 0\} \xleftrightarrow{FT} \{4 \quad -2j \quad 0 \quad 2j\}$$

$$X[0] = \sum_{n=0}^{3} x(n) = 4$$

$$x[0] = \frac{1}{4} \sum_{k=0}^{3} X[k] = 1$$

⑨

Parsevals theorem.

in Fourier series represent Power calc^n.

in Fourier transform represent energy calc.

in Discrete F.T. represent Power calc^n bcz $x(n)$ is periodic.

$$\sum_{n=0}^{N-1} |x[n]|^2 \xleftrightarrow{FT} \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

Proof $\{1\ 2\ 1\ 0\} \xrightarrow{FT} \{4\ \ -2j\ \ 0\ \ 2j\}$

$$\sum_{n=0}^{3} |x(n)|^2 = 6$$

$$\frac{1}{4}\sum_{K=0}^{3} |X[K]|^2 = 6 = \frac{1}{4}\{|16| + |-4| + |0| + |-4|\}$$

$$= \frac{24}{4} = 6$$

(11) ~~2nd op~~ 2nd part of 14 point DFT previous quesn

calculate

ii) $$x[0] = \frac{1}{N}\sum_{K=0}^{N-1} X(K)$$

$$= \frac{1}{14}[X(0) + X(1) \quad \cdots \quad + X(13)]$$

$$= \frac{1}{14}[32 + j0] = \frac{32}{14} = \frac{16}{7}$$

(iii) $x[7] = ?$

concept

when N is even like here $N=14$

$$X\left[\frac{N}{2}\right] = \sum_{n=0}^{N} (-1)^n \cdot x[n]$$

$$x\left[\frac{N}{2}\right] = \frac{1}{N}\sum_{k=0}^{N} (-1)^k$$

~~$x[7] = \frac{1}{N}\sum_{k=0}^{13}$~~

$$x[n] = \frac{1}{N}\sum_{K=0}^{N-1} X[K] \cdot e^{\frac{j2\pi \cdot n \cdot K}{N}}$$

$$x[7] = \frac{1}{N}\sum_{K=0}^{13} X[K] \cdot \underbrace{e^{\frac{j2\pi \cdot 7 \cdot K}{14}}}_{e^{j\pi K}}$$

$$= \frac{1}{N}\sum_{K=0}^{13} X[K] \cdot e^{j\pi K} \longrightarrow \text{for}$$

=

सारे odd point का Σ करके N से divide

Ans: $-\frac{6}{7}$

(ii)

(iii) $\sum_{n=0}^{13} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$

$\Rightarrow \frac{1}{14} [144 + 10 + 25 + 26 + 8 + 45 + 13 + 100 + 13 + 45 + 8 + 26 + 25 + 10]$

$= \frac{1}{14} [144 + 2(10 + 25 + 26 + 8 + 45 + 13) + 100]$

$\frac{498}{14} = \frac{249}{7} = 35.5714$

Q Consider the sequence $x(n)$ for $0 \leq n \leq 11$ and given by

$$x(n) = \{3 \; -1 \; 2 \; 4 \; -3 \; -2 \; 0 \; 1 \; -4 \; 6 \; 2 \; 5\}$$

determine

a) $X[0] = \cancel{13} \cdot \sum_{n=0}^{N-1} x(n) = 13$

b) $X[6] =$

$$X(6) = \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{-j2\pi 6 \cdot n}{12}}$$ — 6 < 12 point DFT

$$X[6] = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\pi n}$$

odd point এ sign change করে add^n.

$$X(6) = -13$$

c) $\sum_{k=0}^{11} X(k) =$

$$x[0] = \frac{1}{N} \sum_{k=0}^{N-1} X(k)$$

$$N \cdot x[0] = \sum_{k=0}^{11} X[k]$$

$$12 \cdot 3 = \underline{\underline{36}}$$

(d) $\sum_{k=0}^{11} |X[k]|^2 = N \sum_{n=0}^{4} |x(n)^2|$

$$= 12\{9+1+4+16+9+4+1+16+36+4+25\}$$

$$= 12\{10+20+10+20+40+25\}$$

$$= \frac{125 \times 12}{\cancel{12}} = 1500$$

Bcz our eqn are lengthy so we introduced a factor W_N

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$W_N^{n.K} = e^{-j\frac{2\pi}{N}.n.K.}$$

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}.K.n} = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nK}$$

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X[K]\, e^{j\frac{2\pi}{N}.n.K} = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot W_N^{-nK}$$

When $n.K = \underbrace{m.N}$
↓
Integer multiple of Fundamental period N

$$W_N^{nK} = 1$$

$$e^{-j\frac{2\pi}{N}.m.N} = e^{-j2\pi m} = 1$$

Q. A length 8 sequence for $0 \leq n \leq 7$ is given by

$x(n) = \{-4 \quad 5 \quad 2 \quad -3 \quad 0 \quad -2 \quad 3 \quad 4\}$ with 8 point DFT $X[K]$.

another sig $y(n)$ with DFT $Y[K]$ is given by

$$Y[K] = W_4^{3K} \cdot X[K]$$

the sequence $y(n)$ will be equal to.

$$Y[k] = W_4^{3K} \cdot X[k]$$

$$Y(k) = e^{-j\frac{2\pi}{4} \cdot 3K} \cdot X[k]$$

$$x(n-n_0) \xrightarrow{FT} e^{-j\frac{2\pi}{N} \cdot k \cdot n_0} X[k]$$

$$Y(k) = e^{-j\frac{2\pi}{8} \cdot 6K} \cdot X[k]$$

$$y[n] = x[n-6]$$

$$y(n) = \{2, -3, 0, -2, 3, 4, -4, 5\}$$

Q Let $X[k]$ denotes the 6 point DFT of real sequence $x(n)$ given by $x[n] = \{1 \ -1 \ 2 \ 3 \ 0 \ 0\}$ another sequence $g(n)$ with DFT $G(K) =$ $G(K) = W_3^{2 \cdot K} X[k]$ The sequence $g(n)$ will be equal to

$$G[k] = e^{-j\frac{2\pi}{3} \cdot 2 \cdot K} X[k]$$

$$G(K) = e^{-j\frac{2\pi}{6} \cdot 4K} X[k]$$

$g[n] = x[n-4]$ → circular right shift of by factor of 4.

$$g(n) = \{2 \ 3 \ 0 \ 0 \ 1 \ -1\}.$$

FFT [Fast Fourier Transform] is a Software algorithm because of which the no. of calc reduces in DFT and DFT becomes very Fast. thats why DFT is serrining till now.

N·DFT

DFT

N-point

(X) multi $\longrightarrow N^2$

(+) Add $\longrightarrow N(N-1)$

Add^n takes min^m time

$(1000)^2 = 10^6$

1 million multiplication required

$= 1000(999)$

9,99,000 Add^n required

FFT (Radix-2)

ω N = 1000

+24 ← adding 24 zeros so that it can be in form of $2^?$.

1024

$\rightarrow \frac{N}{2}\log_2 N = \frac{1024}{2}\log_2 1024$

$= 5120$

$\rightarrow N\cdot\log_2 N = 1024\cdot\log_2^{1024}$

$= 10240$

Q Assume that in a processor, a complex multiplication takes 1 μ sec and that the amount of time to compute DFT is determined by amount of time it takes to perform all the multiplication. How much time does it take to compute 4096 point DFT directly. determine the time if computed by (radix 2) FFT algorithm.

Sol^n

DFT

$X \longrightarrow N^2 = 4096^2 =$

time = 16.777216 sec.

FFT (radix 2)

N = 4096

$\frac{4096}{2}\log_2 4096 = 24576$

time = 24.576 m sec

Digital Filters [don't filter digital signals]

↓

processing of analog s/g using Digital techniques.

D/g Filters are digital techniques for the processing of Analog signals.

How processing done

① First an analog s/g is converted into a dig s/g using A to D converter.
then this converted D/g s/g is processed using digital s/g processor.
and finally the processed digital s/g is converted back into analog s/g. using D to A converters.

There are two types of D/g Filters

FIR [Finite impulse response]	IIR [∞ impulse response]
1. Here the duration of impulse response is finite	1. In IIR, the duration of impulse response is Infinite
2. In these types of systems, since there is no f/b path from o/p to i/p so these systems are also known as non-recursive system.	2. In these type of system, since there is f/b from o/p to i/p so these systems are also known as recursive systems.

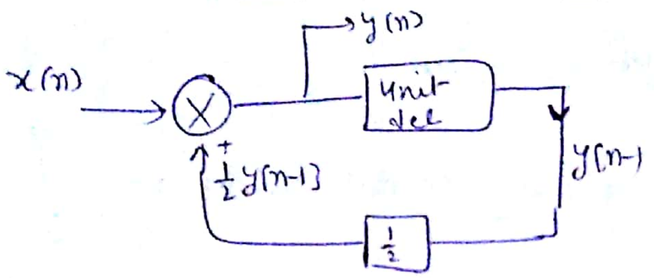
$$h(n) = \{1, 2, 3, 4, 5\}$$

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 5z^{-4} = \frac{Y(z)}{X(z)}$$

$$Y(z) = X(z) + 2z^{-1}X(z) + 3z^{-2}X(z) + 4z^{-3}X(z) + 5z^{-4}X(z)$$

$$y(n) = x(n) + 2x(n-1) + 3x(n-2) + 4x(n-3) + 5x(n-4)$$

↑ o/p only depends upon present i/p and past i/p] not on past o/p.



$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$y(n) = x(n) + \frac{1}{2}y(n-1)$$

Here o/p depends on past o/p i.e $y(n-1)$

FIR guarantee

$$H(z) = 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3} + \frac{5}{z^4}$$

$$\frac{z^4 + 2z^3 + 3z^2 + 4z + 5}{z^4}$$

all poles of FIR at origin. (always)
so Guarteed stable.

FIR is guarnteed stable b/z ~~H(z) At~~ is summable.

3. stability of FIR system is guaranteed.

3. Stability of IIR system can't be guaranteed

Linear phase system → जिस System में o/p कभी Distord हो ही नहीं सकता

Non

4. In FIR systems, the linear phase can be obtained without affecting the stability of system.

4. In IIR system, Linear phase can't be obtained b/z linear phase IIR system are always unstable.

(either we can obtain. Linear phase or Stability)

Disadvantage

5. These systems are very slow systems because to obtain same frequency response as in IIR systems, large no. of multiplication and addition is required. Hence these system can't be used in real time signal processing.

5. These are very fast system and are used when real time sig processing is absolutely essential.

Image processing → FIR

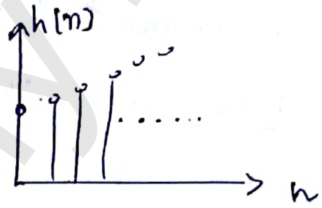
military → IIR

Speech → ~~IIR~~ FIR

6. FIR ~~Filters~~ systems are mainly used in image processing and speech processing

6. IIR systems are mainly used in military applications where real time sig processing is absolutely essential.

stability of IIR is not guaranteed b/c in left page dgm if i change by my $\frac{1}{2}$ factor by 2 then $h(n) = (2)^n u(n)$ so $h(n)$ is unstable system.



Linear phase systems :-

$$x(n) \longrightarrow \boxed{\text{LTI System}} \longrightarrow y[n]$$

$h[n]$

$$H(e^{j\omega}) = e^{-j\omega n_0}$$

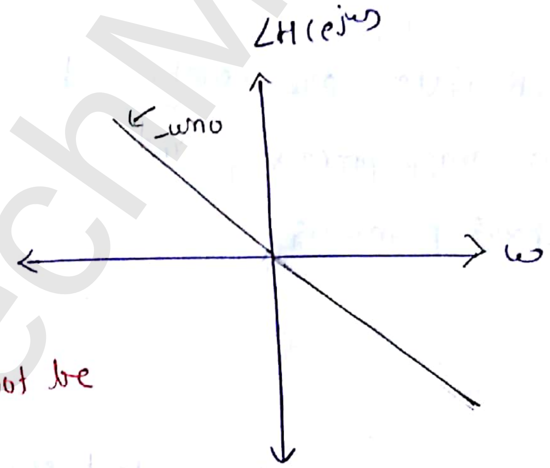
$$y[n] = x[n] * h[n]$$

$$Y(e^{j\omega}) = e^{-j\omega n_0} \cdot X(e^{j\omega})$$

$$\boxed{y[n] = x[\underbrace{n-n_0}_{\text{Delay of } n_0}]}$$

$$\angle H(e^{j\omega}) = -\omega n_0$$

if n_0 is not a const value the $\angle H(e^{j\omega})$ charᶜ wrt ω will not be linear.



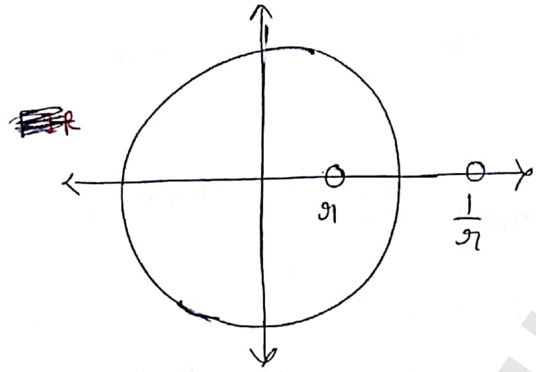
When the phase of the system is a linear funᶜ of frequency, such types of systems are known as <u>linear phase system</u>.

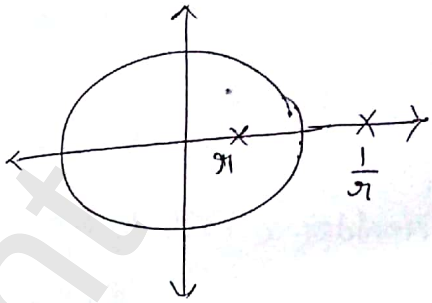
For a linear phase system, since the o/p is always exact replica of input except for constant amount of delay, so there can't be any kind of distortion but if phase is not the linear funᶜ of frqᶜ, the o/p will be always distorted.

Condition for linear phase System :

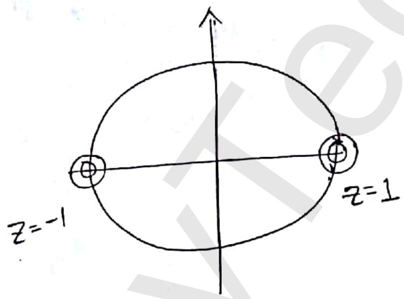
condition 1 .

For linear phase, the real zeros other than unit circle must occur in reciprocal pair i.e





cond^2. Real zero on to the real circle need not be paired bcz it forms its own reciprocal.



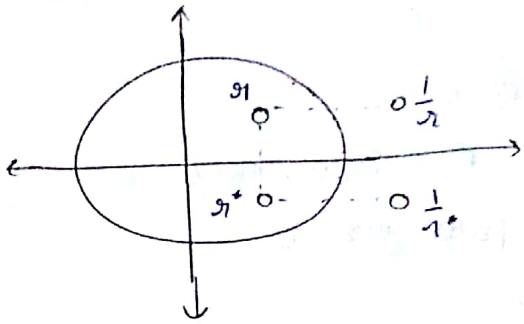
cond^3 complex zero on to the unit circle must occur in conjugate pair i.e if a complex zero is present on to the unit-circle at r another zero must be present at r^*.

$r \longrightarrow r^*$

cond4

complex zeros other than unit circle must occur in conjugate reciprocal quadruple

$r \longrightarrow r^*, \frac{1}{r}, \frac{1}{r^*}$



Q.1 Consider a FIR system with transfer fun^c H(z) given by

$$H(z) = 1 + 2z^{-1} + 2z^{-2} + z^{-3}$$

Checking loc^n of zeros so check linear phase system.

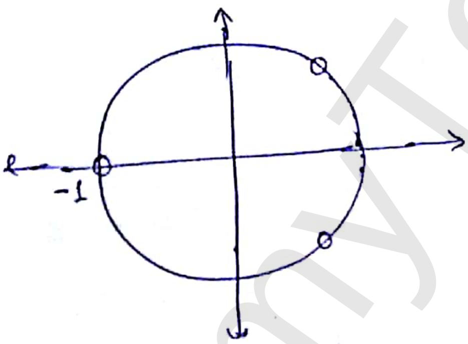
sol^n we have 3 zeros

$z_1 = -1 \longrightarrow$ Real zero on to the unit circle. no need of reciprocal pair.

$z_2 = \frac{1}{2} + j\frac{\sqrt{3}}{2} \longrightarrow$ complex zero at $\sqrt{\frac{1}{4}+\frac{3}{4}} = 1$ unit circle
so cond 3

$z_3 = \frac{1}{2} - j\frac{\sqrt{3}}{2}$

So our system is Linear phase.



method 2

$h(n) = \delta(n) + 2\delta(n-1) + 2\delta(n-2) + \delta(n-3)$

$h(n) = \{1, 2, \vdots 2, 1\}$

$n=1.5$ wrt $n=1.5$ we have even symmetry that's why we are getting cos term.

if wrt n we have odd symmetry we will get sin terms 3.

$H(e^{j\omega}) = 1 + 2e^{-j\omega} + 2e^{-j2\omega} + e^{-j3\omega}$

take 1.5 common { can converted in cosine form }

$= e^{-j1.5\omega}\left[e^{j1.5\omega} + e^{j0.5\omega} + 2 \cdot e^{-j0.5\omega} + e^{-j1.5\omega}\right]$

$H(e^{j\omega}) = e^{-j1.5\omega}\left[\underbrace{2\cos 1.5\omega + 4\cos(0.5\omega)}\right]$

cosine का phase angle 0° होता है।

pair.

$\angle H(e^{j\omega}) = -1.5\omega$

This implies that we have linear phase system.

length of sequence = 4

order of sequence = 3

significance of factor $n=1.5$

$\angle H(e^{j\omega}) = e^{-j\alpha\omega}$

length होगा order से एक ज्यादा होती है

$N = 2\alpha$

$L = 2\alpha + 1$

$\alpha = 1.5$

$N = 3$

$L = 4$

Q2 A 4th order FIR filter has following pairs of zeros

$$z_1, z_2 = 0.5e^{\pm j\pi/6}$$

$$z_3, z_4 = 2e^{\pm j\pi/6}$$

determine whether is system is linear phase or not

Soln:-

$$z_1 = \frac{1}{2}e^{j\pi/6}$$

$$z_1^* = \frac{1}{2}e^{-j\pi/6}$$

$$\frac{1}{z_1} = \frac{1}{\frac{1}{2}e^{j\pi/6}} = 2e^{-j\pi/6}$$

$$\frac{1}{z_1^*} = \frac{1}{\frac{1}{2}e^{-j\pi/6}} = 2e^{j\pi/6}$$

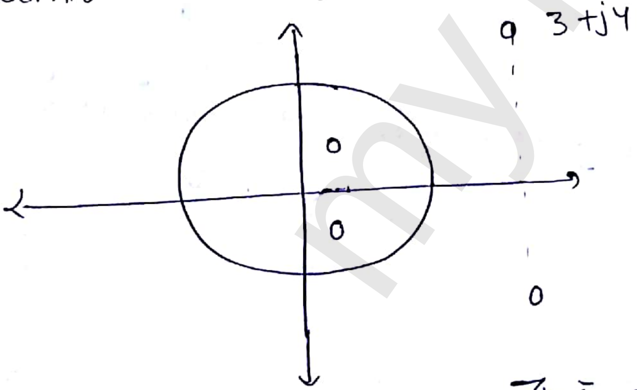
these zeros are matching to z_2, z_3 an z_4

⇒ linear ph-sys

⇒ Linear

Q3 For a fourth order FIR filter, the locn of one complex zero is given by $z_1 = 3 - j4$ for the system to be a linear phase system determine the locn of other zero.

Soln



$$z_4 = \frac{1}{3+j4} = \frac{3-j4}{(3+j4)(3-j4)} = \frac{3}{25} - j\frac{4}{25}$$

$$z_1 = 3 - j4$$

$$z_2 = z_1^* = 3 - j4$$

$$z_3 = \frac{1}{z_1} = \frac{1}{3-j4} = \frac{(3+j4)}{9+16} = \frac{3}{25} + j\frac{4}{25}$$

Q4 A causal LTI FIR discrete time System is characterised by an impulse response $h[n] = a_1 \delta(n) + a_2 \delta(n-1) + a_3 \delta(n-2) + a_4 \delta(n-3)$

$$+ a_5 \delta[n-4] + a_6 \delta[n-5] + a_7 \delta[n-6]$$

For what values of impulse response samples, will its freq response $H(e^{j\omega})$ have a linear phase

$\cancel{h[n]} = \{ a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \}$

$n=3$

Ans: $\boxed{a_7 = -a_1 \qquad a_6 = -a_2 \qquad a_5 = -a_3}$

Length	Symmetry	Type	Location of zeros
even	even	II	→ even no. of zeros or no zero is present at $z=1$ & odd no. of zero are present at $z=-1$
even	odd	IV	→ even no. of zeros or no zero is present at $z=-1$ & odd no. of zero are present at $z=1$
odd	even	I	→ even no. of zeros or no zero are present at $z=\pm 1$
odd	odd	III	→ odd no. of zeros are present at $z=\pm 1$

II → even no. of zeros or No zero is present at $z=1$ & odd no. of zeros are present at $z=-1$

IV – " " " " " " " " " " $z=-1$ " " " " " " , $z=+1$

I – " " " " " " " are " " $z=\pm 1$

III – odd " " " are present at $z=\pm 1$

L	S					
1	1	2	E	$z=+1$	O	$z=-1$
1	0	4	E	$z=-1$	O	$z=+1$
0	1	1	E	$z=\pm 1$		
0	0	3			O	$z=\pm 1$

Q.1 A length 10, type 2 real coefficient FIR filter has the following zeros $z_1 = 3$ $z_2 = j0.8$ $z_3 = j$ determine the locⁿ of remaining zeros.

Solⁿ length = 10

mean order = 9

means 9 zeros are present

FIR means linear phase system

$z_1 = 3 \longrightarrow z_4 = \frac{1}{3}$

$z_2 = j0.8 \longrightarrow$ $z_5 = -j0.8$

$z_6 = \frac{1}{j0.8} \times \frac{j}{j} = -j1.25$

$z_7 = j1.25$

$z_3 = j \longrightarrow z_8 = -j$

now go to type 2.

So $z_9 = -1$

Q A length 10 type 4 Real coefficient FIR filter has the following zeros

$z_1 = -1.2 + j1.4$

$z_2 = \frac{1}{2} + j\frac{\sqrt{3}}{2}$

$z_3 = \frac{1}{4} + j\frac{\sqrt{15}}{4}$

Soln length = 10

so ~~zeros~~ order = 9

so zeros = 9

FIR filter means ⇒ linear phase system.

$z_1 = -1.2 + j1.4 \longrightarrow z_4 = -0.35 - j0.4$

$z_5 = -1.2 - j1.4$

$z_6 = -0.35 + j0.4$

→ present at unit circle

$z_2 = \frac{1}{2} + j\frac{\sqrt{3}}{2} \longrightarrow z_7 = +\frac{1}{2} - j\frac{\sqrt{3}}{2}$

→ zeros at unit circle.

$z_3 = \frac{1}{4} + j\frac{\sqrt{15}}{4} \longrightarrow z_8 = \frac{1}{4} - j\frac{\sqrt{15}}{4}$

Now type IV

$z_9 = 1$

Q A length 9 type 1 real coefficient FIR filter has the following zeros $z_1 = -0.5$

$z_2 = 0.3 + j0.5$

$z_3 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

determine the no. of remaining zeros.

Soln length = 9 = odd

order = 8

No. of zeros = 8

FIR filter means ⇒ linear phase filter

$z_1 = -0.5$

$z_2 = 0.3 + j0.5$

$z_3 = -\frac{1.2}{2} + j\frac{\sqrt{3}}{2}$

$z_4 = -2$

$z_5 = 0.3 - j0.5$

$z_6 = 0.88 - j1.47$

$z_7 = 0.88 + j1.47$

$z_8 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

this correct

$z_6 = 0.12 - j0.1993$

$z_7 = 0.12 + j0.1993$

$\sqrt{(1.2)^2 + \frac{3}{4}}$

Q A length 13 type 3 Real coefficient FIR filter has the following zeros

$z_1 = -0.3 + j0.5$

$z_2 = j0.8$

$z_3 = -0.3$

remaining zeros ?

Sol^n length = 13

order = no. of zeros = 12

$z_1 = -0.3 + j0.5$

$z_4 = -0.3 - j0.5$

$z_5 = -0.88 - 1.47j$

$z_6 = -0.88 + 1.47j$

$z_2 = j0.8$

$z_7 = -j0.8$

$z_8 = \frac{1}{j0.8} \times \frac{j}{j} = -j1.25$

$z_9 = \frac{1}{-j0.8} \times \frac{j}{j} = j1.25$

$z_3 = -0.3$

$z_{10} = -3.33$

type 3

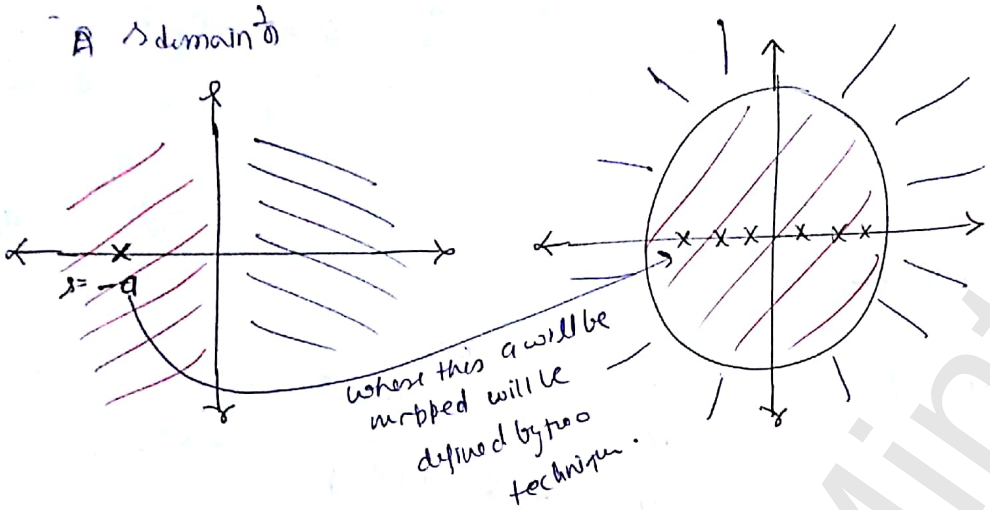
$z_{11} = +1$

$z_{12} = -1$

IIR filter design (These filters are not designed directly)

Analog filter को d/g filter में convert करके देते हैं।

A S domain में



① Impulse invariance technique

Q For the analog transfer fun^c $H(s) = \frac{2}{(s+1)(s+2)}$, determine digital transfer fun^c $H(z)$ using impulse invariance method assuming sampling time $T_s = 1$ sec.

Sol^n

$$\frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$T_s = 1$ sec

$$\frac{2}{(s+1)} - \frac{2}{(s+2)}$$

$$h(t) = 2e^{-t}u(t) - 2e^{-2t}u(t)$$

| sample $h(t)$ into $h[n]$

$$h(t) \longrightarrow h[n]$$

| in general we do ↓ but actually
$x(t) \longrightarrow x[n]$
$x[n \cdot T_s]$

हम nT_s नहीं लिखते हैं

$$h[n] = 2e^{-nT_s}u[n] - 2e^{-2 \cdot nT_s}u[n]$$

$$h[n] = 2e^{-n}u(n) - 2e^{-2n}u[n]$$

$$H(z) = \frac{2}{1-e^{-1}z^{-1}} - \frac{2}{1-e^{-2}z^{-2}}$$

this is the simple method of conversion of H(s) into H(z)

$$H(s) \longrightarrow h(t) \xrightarrow{\text{sample}} h(n) \longrightarrow H(Z)$$

s domain

Q.2 $H(s) = \frac{10}{s^2+8s+15}$ $T_s = 0.1$ sec $H(z) = \frac{Az}{z^2-Bz+C}$

T

$$\frac{-8\pm\sqrt{\quad 10}}{s^2+5s+3s+15}$$

$$\frac{10}{(s+5)(s+3)} = \frac{A}{(s+5)} + \frac{B}{(s+3)} = \frac{-5}{s+5} + \frac{5}{(s+3)}$$

$$h(t) = -5e^{-5t}u(t) + 5e^{-3t}u(t)$$

$$h[n] = -5e^{-5.n.T_s}u(n) + 5e^{-3nT_s}u(n)$$

$$= -5e^{-5n}u(n) + 5e^{-0.3n}u(n)$$

$$= \left(\frac{1}{8}\right)$$

$$= -5\left(\frac{1}{e^{.5}}\right)^n u(n) + 5\left(\frac{1}{e^{0.3}}\right)^n u(n)$$

$$= -5\left(\frac{z}{z-\cancel{1.65}\ 0.61}\right) + 5\left(\frac{z}{z-\cancel{1.35}\ 0.74}\right)$$

$$= \frac{-5z}{(z-\cancel{1.65})_{0.61}} + \frac{5z}{(z-\cancel{1.35})_{0.74}}$$

$A = .675$

$B = 1.347$

$C = 0.45$

$$\cancel{-5z(z-1.35) + 5z(z-1.65)}$$

$$\cancel{5z\{-z+1.35+z-1.65\}}$$

$$= \frac{\cancel{-1.5z}}{\cancel{(z-1.65)(z-1.35)}}$$

$$= \frac{-5z(z-0.74)+5z(z-0.61)}{(z-0.61)(z-0.74)}$$

$$\frac{-\cancel{5z^2} + z\,3.7 + \cancel{5z^2} - 3.05z}{z^2 - 1.35z + 0.45} = \frac{0.65z}{z^2 - 1.35z + 0.45}$$

$$\frac{0.65z}{z^2-1.35z+0.45} = \frac{Az}{z^2-Bz+C}$$

$A = 0.65$

$B = 1.35$

$C = 0.45$

II Bilinear Transformation (BLT):-

$$\boxed{s = \frac{2}{T_s} \cdot \frac{1-z^{-1}}{1+z^{-1}}}$$

$$\boxed{s = \frac{2}{T_s} \frac{z-1}{z+1}}$$

Q Consider an analog transfer funᶜ $H(s) = \frac{1}{s+2}$; $T_s = 1$ sec

determine $H(z)$ using ~~DF~~ BLT

Solⁿ $H(z)\Big|_{s = 2 \cdot \frac{z-1}{z+1}} = \frac{1}{2\frac{(z-1)}{(z+1)} + 2} = \frac{(z+1)}{2z-2+2z+2}$

$= \frac{(z+1)}{4z}$ Ans

Q $H(s) = \frac{2}{(s+5)(s+7)}$; $T_s = 1$ sec determine $H(z)$

Solⁿ

$H(z) =$

$s = 2\left(\frac{z-1}{z+1}\right) = \frac{2(1-z^{-1})}{(1+z^{-1})}$

$\frac{2}{\frac{2z-2}{z+1}}$

$\frac{2}{\left(\frac{2-2z^{-1}}{1+z^{-1}} + 5\right)\left(\frac{2-2z^{-1}}{1+z^{-1}} + 7\right)}$

$= \frac{2(1+z^{-1})^2}{(2-2z^{-1}+5+5z^{-1})(2-2z^{-1}+7+7z^{-1})}$

$= \frac{2(1+z^{-1})^2}{(7+3z^{-1})(9+5z^{-1})} = \frac{2(1+z^{-2}+2z^{-1})}{63+35z^{-1}+27z^{-1}+15z^{-2}} =$

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{7.5z^{-2} + 31z^{-1} + 31.5} \quad // \text{Ans}$$

Amplitude modulation

$$X_{AM}(t) = [A_c + m(t)] \cos \omega_c t$$

$$X_{DSB}(t) = m(t) \cdot c(t)$$

$$X_{SSB}(t) = m(t) \cdot \cos \omega_c t \pm \hat{m}(t) \sin \omega_c t$$

$$X_{AM}(t) = A_c \cos \omega_c t + \frac{m A_c}{2} \cos(\omega_c + \omega_m) t + \frac{m A_c}{2} \cos(\omega_c - \omega_m) t$$

$$\boxed{m = \frac{A_m}{A_c}}$$

$$P_t = P_c \left[1 + \frac{m^2}{2}\right]$$

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

$$\eta = \frac{P_{SB}}{P_t} \times 100 \%$$

$$\eta = \frac{m^2}{2 + m^2} \times 100 \%$$

① The transmission B.W for full AM signal as well as DSBSC signal is $2f_m$ in H_z and $2\omega_m$ in rad/sec where $f_m = \omega_m$ is the maximum modulating sig frq.

② For SSB-SC modulation, the transmission BW is f_m (ω_m).

Q1 The O/P sig from an AM modulator is given by $x(t) = 5\cos 1800\pi t + 20\cos 2000\pi t + 5\cos(2200\pi t)$ determine

① carrier sig $c(t)$ and modulation index m.

$$5\cos 1800\pi t + 20\cos 2000\pi t + 5\cos(2200\pi t)$$

$5\cos 1800\pi t$ → lower band; $20\cos 2000\pi t$ → Midd frq^c is equal to carrier; $5\cos(2200\pi t)$ → High frq^c

Sol^n

carrier sig → $c(t) = 20\cos 2000\pi t$

(20 → A_c, 2000π → ω_c)

$$\frac{mA_c}{2} = 5 \quad \{\text{given}\}$$

$$mA_c = 10$$

$$m = \frac{10}{20} = 0.5$$

ⓑ determine modulating sig $m(t)$

$$m(t) = A_m \cos \omega_m t$$

$$m = \frac{A_m}{A_c} \Rightarrow 0.5 = \frac{A_m}{20} =$$

$$\boxed{A_m = 10}$$

$$\omega_c + \omega_m = 2200\pi$$

$$2000\pi + \omega_m = 2200\pi$$

$$\boxed{\omega_m = 200\pi}$$

ⓒ determine the ratio of Power in side bands to the power in carrier.

$$P_t = P_c\left[1 + \frac{m^2}{2}\right]$$

$$P_{total} = \underbrace{P_c}_{\text{Carrier Power}} + \underbrace{P_c\frac{m^2}{2}}_{\text{sideband power } P_{SB}}$$

$$\frac{P_{SB}}{P_c} = \frac{P_c \frac{m^2}{2}}{P_c} = \left(\frac{m^2}{2}\right)$$

$$= \frac{(0.5)^2}{2} = 0.125$$

Q.2 an AM modulator has the o/p given by

$x(t) = A\cos 2\pi 200t + B\cos 2\pi 180t + B\cos 2\pi \cdot 220t$.

the carrier power is 100 watt and $P_t = 40\%$. determine the values of A and B.

Sol^n $\underbrace{A}\cos (2\pi 200)t + B\cos 2\pi 180t + B\cos 2\pi 220t$

$A \rightarrow A_c$, $2\pi 200 \rightarrow \omega_c$

$P_c = \frac{A^2}{2}$

$\sqrt{2\times 100} = A^2$

$\sqrt{200} = A = 14.142 = A_c$

$\boxed{B = 8.16}$

$\frac{m A_c}{2} = B$

$m \times 14.14 = 2B$

~~$P_t = P_c\left[1 + \frac{m^2}{2}\right]$~~

~~$\frac{0.40}{100} = P_c\left[1 + \frac{m^2}{2}\right]$~~

$\eta = \frac{m^2}{2+m^2} \times 100\%$

$0.4 = \frac{m^2}{2+m^2}$

find m

then find B.

Q For an Am sig the transmission B/w is 10kHz if the Highest f_m c component present in 705 KHz then the value of carrier f_m c will be

Soln

$$2 f_m = 10 \text{ kHz}$$

$$f_m = 5 \text{ kHz}$$

$f_c - f_m$ → Lowest f_m component

f_c → middle component

$f_c + f_m = 705$ kHz → Highest f_m component

Q A certain Am transmitter radiates 10kW with the carrier unmodulated and 11.8 kW when the carrier is sinusoidally modulated. determine modulation index m.

(b) if another sin wave corresponding to 30% modulation if transmitted simultaneously then determine the total radiated power.

Soln

$$\frac{A_m^2}{2} = 10$$

$$\frac{A_m^2}{2} + \frac{A_c^2}{2} = 11.8$$

$$\frac{A_c^2}{2} = 1.8$$

$$m = \frac{A_m}{A_c} = \frac{\sqrt{20}}{\sqrt{3.6}}$$

$$m = 2.357 \times$$

concept

$$m_T^2 = m_1^2 + m_2$$

(a) $P_t = 11.8\,kW$

$P_c = 10\,kW$

$1 + \frac{m^2}{2} = 1.18$

$\frac{m^2}{2} = 0.18$

$m^2 = 0.36$

$m = 0.6$

Concept

(b) $\boxed{m_T^2 = m_1^2 + m_2^2 + m_3^2 + \cdots}$

$m_1 = 0.6$

$m_2 = 0.3$

$m_T^2 = 0.36 + 0.09 = 0.45$

$P_t = 10\,kW\left(1 + \frac{0.45}{2}\right) = 12.25\,kW$

Q. the o/p current of 50% modulated AM generator is 1.8 Amp to what value will this current rise if the gen^r is modulated additionally by another sine wave whose modulation index is 0.6

Sol^n $m = 0.5$

$I_t = I_c\sqrt{1 + \frac{m^2}{2}}$

$1.8 = I_c\sqrt{1 + .125}$

$I_c = 1.69705$

$m_T^2 = 0.5^2 + 0.6^2 = \cancel{0.78}\ 0.61$

$I_t = 1.7\sqrt{1 + \frac{.61}{2}}$

$I_t = 1.94$ Amp.

(b) What will be the % power saving if the carrier and one of the ~~carrier~~ side band are suppressed. (i.e single side band suppressed carrier)

Concepts

DSBSC $P_t = P_c + P_c \frac{m^2}{2}$

$$\%\text{age Power Saving} = \frac{\text{Total Power Saved}}{\text{Total Tx. Power}} \times 100$$

$$\boxed{\%\text{ Power Saving} = \frac{P_c}{\left(P_c + P_c \frac{m^2}{2}\right)} = \frac{P_c}{P_c\left[1+\frac{m^2}{2}\right]} = \frac{2}{2+m^2} \times 100\%}$$

For SSBSC

$$P_t = P_c + \frac{P_c m^2}{4} + \frac{P_c m^2}{4}$$

$$\%\text{ Saving in Power} = \frac{\cancel{P_c}\left[1+\frac{m^2}{4}\right]}{\cancel{P_c}\left[1+\frac{m^2}{2}\right]} = \frac{1}{4} \frac{(4+m^2) \cdot 2}{(2+m^2)}$$

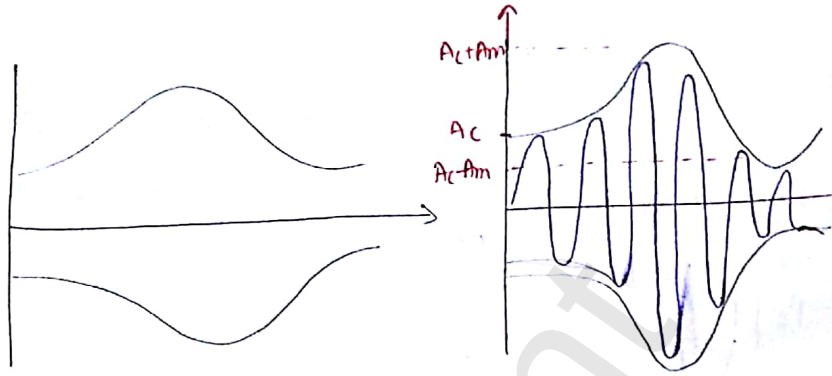
$$\boxed{\%\text{ Saving} = \frac{1}{2} \frac{(4+m^2)}{(2+m^2)} = \frac{4+m^2}{(4+2m^2)} \times 100}$$

$$\%\text{age Saving} = \frac{4+0.61}{4+2\times 0.61} = \frac{4.61}{5.22} = 88.31\%$$

this much of Power is saved

Q A given AM broadcast station transmitts an average carrier Power o/p of 40KW and uses a modulation index of 0.707 for sine wave modulation. What will be the peak amplitude of o/p if the antenna is represented by 50Ω resistive load.

Soln



$m = 0.707$

$P_c = 40KW$

$P_t = P_c\left(1 - \frac{m^2}{2}\right)$

$P_t = 0.75 \times 40 = 30KW$

$m = \frac{A_c}{A_m}$

$0.707 = \frac{\sqrt{80}}{A_m}$ $\quad A_m = 12.651$

$A_m + A_c = 21.59$

using

$\boxed{40 = \frac{A_c^2}{2R}}$

$A_c = \sqrt{80R}$

$A_c = \sqrt{80 \times 50}$

By Sir

$P_c = 40KW = \frac{A_c^2}{2 \cdot R}$ → using this formula. bcz ~~actual~~ Resistance is given so this is not normalized power this is actual power

$A_c^2 = 4 \times 10^4 \times 10^2$

$A_c = 2000\ V$

$m = 0.707$

$A_m = m \cdot A_c = 0.707 \times 2000 = 1414\ V$

$A_{max} = A_c + A_m = 3.414 KV$

Q. An AM modulator produces 24 kW power o/p when modulated to 100%. Now the modulation is reduced to 30% and after modulation, single side band with carrier power reduced by 26 dB is transmitted. determine total o/p power.

$m = 1$

$P_t = P_c\left[1 + \frac{m^2}{2}\right]$

$24 = P_c\left[1 + \frac{1}{2}\right]$

$24 = P_c \times \frac{3}{2}$

$\frac{24 \times 2}{3} = P_c = 16\,KW$

$\boxed{P_c = 16\,KW}$

$m_{New} = 0.7$

this should be reduced by 26db

$= P_c + \frac{P_c m^2}{4}$

$+ \cancel{16} \times \frac{(0.7)^2}{4}$

By Sir

$P_c = 16\,KW$

$P_t = 16\,KW\left[1 + \frac{0.09}{2}\right] = 16.72\,K$

$P_{SB} = 16 \times 10^3 \times \frac{0.09}{2} \times \frac{1}{2} = 360\,W$

reduced by 26dB

$(P_c)_{dB} = 10 \log_{10} 16 \times 10^3 = 42.04\,dB - 26\,dB$

$= 16.04\,dB$

$P_c = 10^{1.604} = 40.2\,W$

$P_t = P_{SB} + P_c$

$= 360W + 40.2W$

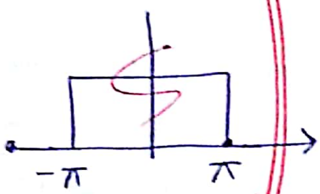
$= 400.2W$

Q In a DSB-SC system, the carrier is $c(t) = A\cos 2\pi f_c t$ and the message sig is given by $m(t) = \text{Sinc}\,t + \text{Sinc}^2 t$ the BW of modulated sig in Hz will be

Soln

$m(t) = \text{Sinc}\,t + \text{Sinc}^2 t$

$A\ 1 \cdot Sa(\pi t) + 1 \cdot Sa^2(\pi t)$



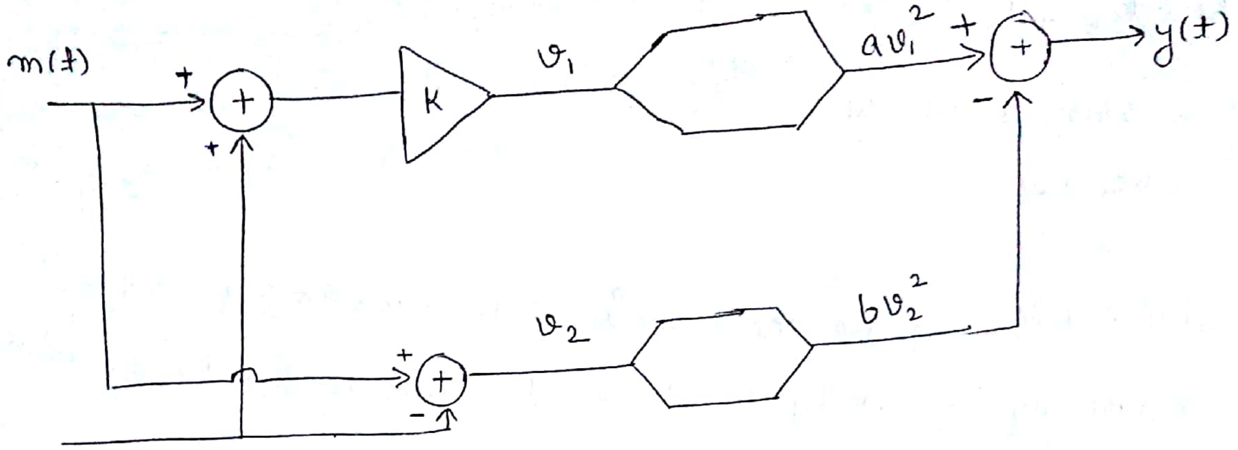
max^m f_m $= \pi$

max^m f_m $= 2\pi$

$B \cdot W = 2 \cdot 2\pi$ rad/sec $= 4\pi$ rad/sec

$B \cdot W = 2$ Hz

Q Consider a modulation system as shown below



$C(t) = A\cos\omega_c t$

the system is used to generate the DSB-SC signal without filtering the system is used to generate the DSB-SC s/g. determine the value of gain K. so that o/p of the system is DSB-SC s/g.

Sol^n

$$v_1 = (m(t) + A\cos\omega_c t)K$$

$$v_2 = (m(t) - A\cos\omega_c t)$$

$$y(t) = aK^2\{m(t) + A\cos\omega_c t\}^2 - b\{m(t) - A\cos\omega_c t\}^2$$

$$y(t) = K^2\{a\,m^2(t) + aA^2\cos^2\omega_c t + a2(m(t)A\cos\omega_c t)\} - b\,m(t)^2 - A^2\cos^2\omega_c t + b2\,m(t)\cos\omega_c t\}$$

~~$K^2\{am^2(t) + aA^2\cos^2\omega_c t - b\,m(t)^2 - A^2\cos^2\omega_c t$~~

$$aK^2\,2m(t)\,A\cos\omega_c t + b2m(t)\cos\omega_c(t)\ \text{so}$$

$$aK^2 = -b$$

$$K = \sqrt{\frac{-b}{a}}$$

by sir

$$v_1 = k[m(t) + c(t)]$$

$$a \cdot v_1^2 = a k^2 [m^2(t) + c^2(t) + 2m(t)c(t)]$$

$$b \cdot v_1^2 = b \cdot [m^2(t) + c^2(t) - 2m(t) \cdot c(t)]$$

$$y(t) = (m^2(t) + c^2(t))[ak^2 - b]$$

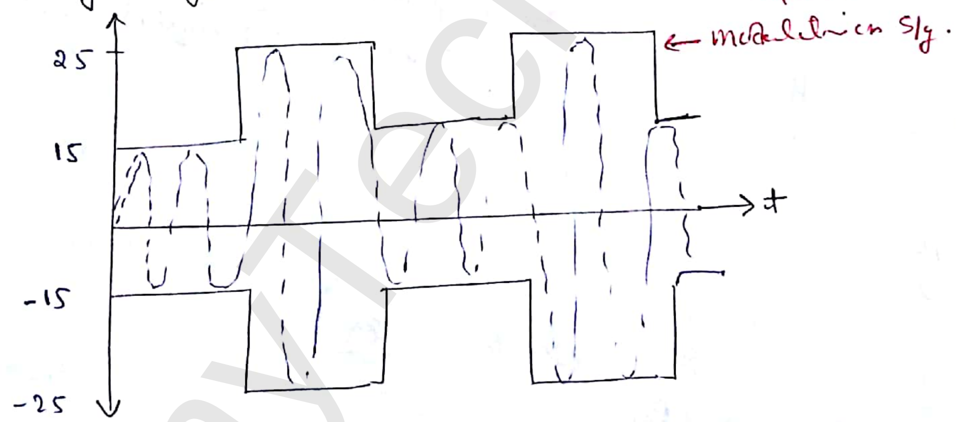
$$= \cancel{2(mt)}\ 2\{m(t) \cdot c(t)\}\ (ak^2+b)$$

Beauty of this system is that we can remove unwanted sig without using filter. just by making $K = \sqrt{\frac{b}{a}}$

$$(a^2k^2 - b) = 0$$

$$k = \sqrt{+\frac{b}{a}}$$

Q a periodic sig modulates a High freq carrier sig to produce the following waveforms



determine ① modulation index

② Power η

③ is it possible to envelope detect the message sig from the modulated sig.

To detect Am two method

$A_c + A_m = 25$

$A_c - A_m = 15$

$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{25-15}{25+15} = \frac{10}{40} = \frac{1}{4}$$

$$\eta = \frac{m^2}{2+m^2} \times 100 \Big]$$ this formula is for ~ modulating sig. but here ⊓⊔ modulat sig

$$\eta = \frac{\text{useful Power}}{P_t} \times 100\,\%$$

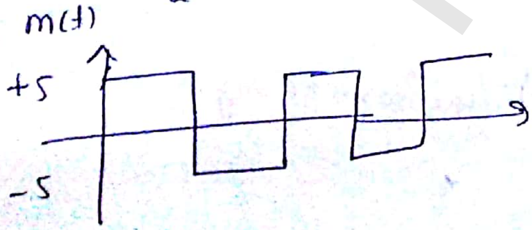
$$x_{Am}(t) = \underbrace{A_c \cos\omega_c t}_{P_c} + \underbrace{m(t)}\cdot\underbrace{\cos\omega_c t}_{P_{SB}}$$

agar दो periodic sig को multiply करते हैं तो उनकी Power भी multiply हो जाती है।

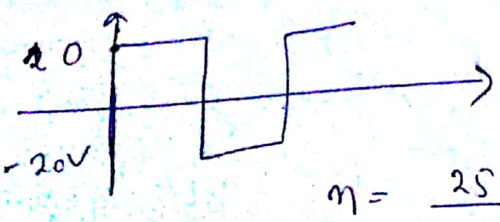
$$P_{SB} = P_m \cdot \frac{1}{2} = \frac{P_m}{2}$$

$$P_c = \frac{A_c^2}{2}$$

$$\eta = \frac{\frac{P_m}{2}}{\frac{A_c^2}{2} + \frac{P_m}{2}} = \frac{P_m}{A_c^2 + P_m} \times 100\,\% =$$



$P_m = (A_m)^2 = 25W$



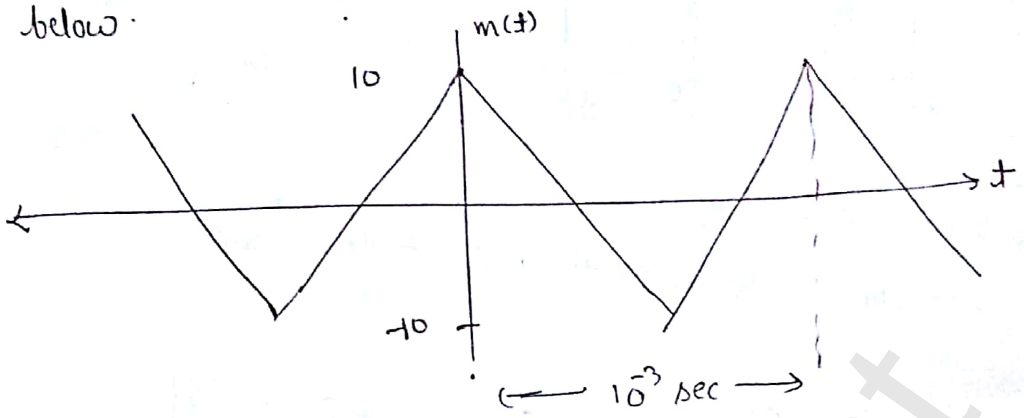
$(A_c^2) = 400$

$$\eta = \frac{25}{400+25} = \frac{25}{425} \times 100 = \quad \%$$

Q Consider an AM sig given by

$$x(t) = [A + m(t)] \cos \omega_c t$$

Here the modulating sig m(t) is a periodic triangular wave as shown below.



for m = 0.8, determine

① Amplitude and power of carrier

② Sideband power and power efficiency η.

Soln

$A_m = 10$

$m = \frac{A_m}{A_c}$

$0.8 = \frac{10}{A_c}$

$A_c = \frac{100}{8} = 12.5$

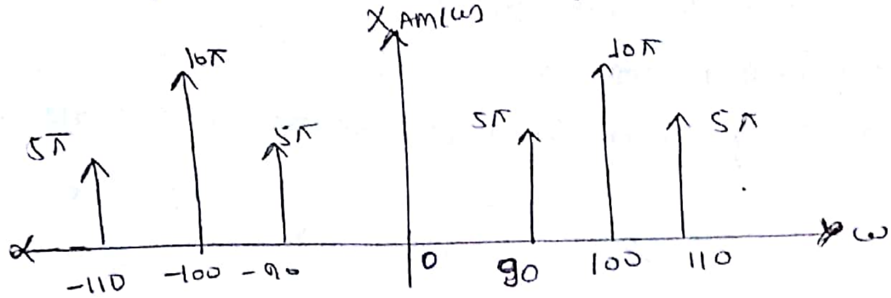
carrier Power = $\frac{(12.5)^2}{2} =$ ~~52.08~~ 78.125 watt

Side band Power (P_{SB}) = $\frac{P_m}{2} = \frac{100/3}{2} = \frac{100}{6} = 16.67$ watt

$P_m = \frac{(10)^2}{3} = \frac{100}{3}$

$$\eta = \frac{100/6}{78.125 + 100/6} = 17.58\%$$

Q The spectrum of a sinosoidal AM s/g is shown below



① determine modulation index m

② determine weather the s/g is envelope detectable or not

Sol^n

$$A_c \cos\omega_c t \xleftrightarrow{FT} A_c \cdot \pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

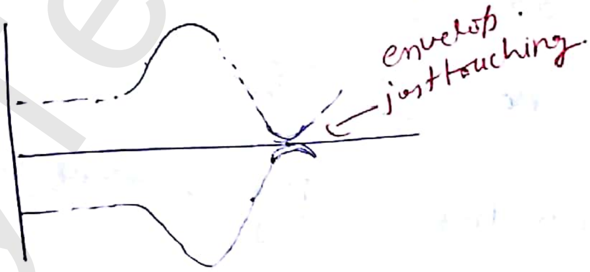
$$A_c\pi = 10\pi$$

$$A_c = 10$$

$$\frac{m A_c}{2} \pi = 5\pi$$

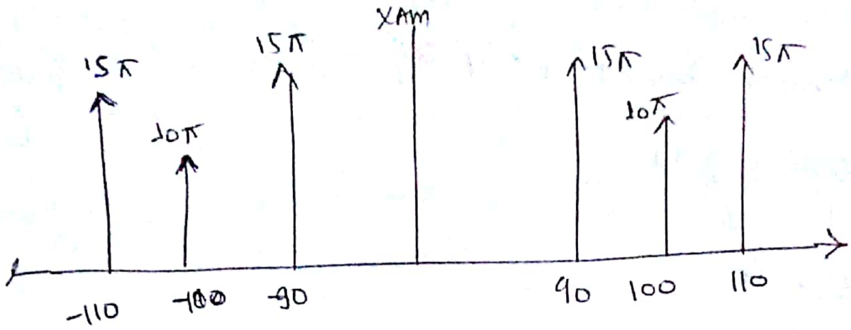
$$m = \frac{10}{A_c} = \frac{10}{10} = 1$$

$m=1$ means



$$x_{AM}(t) = A_c \cos\omega_c t + \frac{m A_c}{2} \cos(\omega_c + \omega_m) t$$
$$+ \frac{m A_c}{2} \cos(\omega_c - \omega_m) t$$

Q.



(a) find m.

$$A_c \pi = 10\pi$$

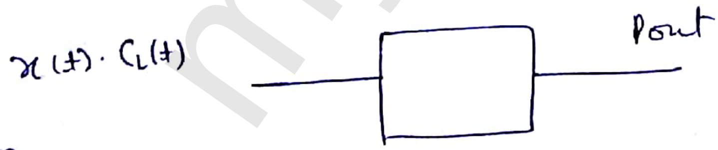
$$A_c = 10$$

$$\frac{mA_c}{2}\pi = 15\pi$$

$$m = \frac{15 \times 2}{10} = 3$$

envelope detection not possible
but by another method synchr detection method we can detect

Q A DSB-SC sig $x(t) = A\,m(t)\cos\omega_c t$ is multiplied with a local carrier $C_L(t) = \cos(\omega_c t + \theta)$ and the o/p is passed through a ideal low pass filter with a cutoff frq equal to BW of msg sig m(t). denoting the Power of sig at the o/p of low pass filter by Pout and power of modulated sig by Pin then the ratio Pout/Pin for $\theta = \pi/4$ will be



$$x(t)\cdot C_L(t) = A\,m(t)\cdot\cos\omega_c t \cdot \cos(\omega_c t + \theta)$$

$$= A\,m(t)\{\cos\omega_c t\}\cdot\{\cos\omega_c t\cos\theta - \sin\omega_c t\sin\theta\}$$

$$= A\,m(t)\cos^2\omega_c t\cdot\cos\theta - \frac{A\,m(t)}{2}\sin 2\omega_c t\cdot\sin\theta$$

By Sir

$x(t) = A\, m(t) \cdot \cos\omega_c t$ ← i/p sig

$= A \cos\omega_c t \; m(t)$

$$P_{in} = \frac{A^2}{2} \cdot P_m$$

$$x(t) \cdot c_L(t) = A\, m(t) \cdot \cos(\omega_c t + \theta) \cdot \cos\omega_c t$$

→ this sig will not pass through filter

$$= \frac{A}{2}\, m(t) \left[\cos(2\omega_c t + \theta) + \cos\theta\right]$$

o/p sig. $\simeq$ $= \frac{A}{2}\, m(t) \cdot \cos\theta$

$$y(t) = \frac{A}{2}\, m(t) \cdot \cos\theta$$

$$y(t) = \frac{A}{2\sqrt{2}} \cdot m(t)$$

$$P_{out} = \frac{A^2}{8} \cdot P_m$$

$$\frac{P_{out}}{P_{in}} = \frac{\frac{A^2}{8} \cdot P_m}{\frac{A^2}{2} \cdot P_m} = \frac{1}{4}$$

Superheterodyne receiver

In superheterodyne receiver the image frequency is given by

$$\rightarrow \underbrace{f_{si}}_{\text{Image frq}} = \underbrace{f_s}_{\text{signal frq}} + 2 \cdot IF$$

IF ↳ intermediate frq = 455 KHz (in India)

Q A super heterodyne receiver uses an IF frq of 455 KHz. If it is tuned at 1120 KHz then the value of image sig frq will be

Soln

$f_s = 1120$

$f_{si} = 1120 + 2(455)$

$f_{si} = 1120 + 910$

$f_{si} = 2030$ KHz