Date 14 Aug

EMFT

Coulombs Force Law — $\vec{F_{12}}$ is force on Q_2 charge located at P_2 due to Q_1 charge located at P_1

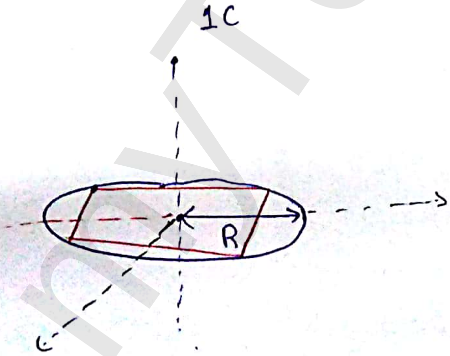
$$\boxed{\vec{F_{12}} = -\vec{F_{21}}}$$

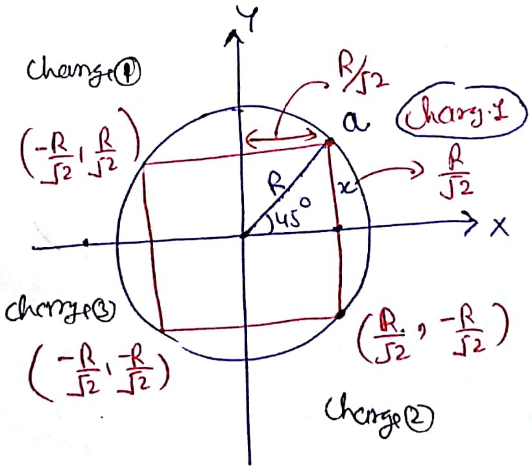
$$R_{12} = P_2 - P_1$$

force locn − Source locn

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon |R_{12}|^3} \vec{R_{12}}$$

Q Find the force on 1C charge located at R height on the axis of the ring (having a R radius) of four charges placed symmetrically at 90° to each other each charge is Q coulomb.



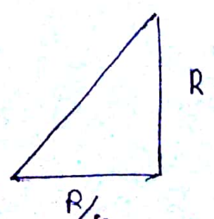


$$F = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$\vec{F} = \frac{Q(1)}{4\pi\epsilon}$$

$$\sin \cancel{\cos} 45^\circ = \frac{x}{R}$$

$$\frac{R}{\sqrt2} = x$$



$$\sqrt{\frac{R^2}{2} + R^2} = \sqrt{\frac{3}{2}} R$$

$$\vec{F}_1 = \frac{Q(1)}{4\pi\epsilon} \frac{\left(0-\frac{R}{\sqrt{2}}\right)\hat{a}_x + \left(0-\frac{R}{\sqrt{2}}\right)\hat{a}_y + (R-0)\hat{a}_z}{\left(\sqrt{\frac{R^2}{2}+\frac{R^2}{2}+R^2}\right)^3}$$

$$\vec{F}_2 = \frac{Q(1)}{4\pi\epsilon} \frac{\left(0-\frac{R}{\sqrt{2}}\right)\hat{a}_x + \left(0+\frac{R}{\sqrt{2}}\right)\hat{a}_y + (R-0)\hat{a}_z}{\left(\sqrt{\frac{R^2}{2}+\frac{R^2}{2}+R^2}\right)^3}$$

$$\left(\sqrt{\frac{4R^2}{2}}\right)^3$$

$$\left(\frac{2R}{\sqrt{2}}\right)^3 = (\sqrt{2}R)^3$$

$\cancel{\vec{F}_3} =$

$$\vec{F}_3 = \frac{Q(1)}{4\pi\epsilon} \frac{\left(0+\frac{R}{\sqrt{2}}\right)\hat{a}_x + \left(0+\frac{R}{\sqrt{2}}\right)\hat{a}_y + (R-0)\hat{a}_z}{\left(\sqrt{}\right)^3}$$

$$\vec{F}_4 = \frac{Q(1)}{4\pi\epsilon} \frac{\left(0+\frac{R}{\sqrt{2}}\right)\hat{a}_x + \left(0-\frac{R}{\sqrt{2}}\right)\hat{a}_y + (R-0)\hat{a}_z}{\left(\sqrt{}\right)^3}$$

$$F = \bar{F}_1 + F_2 + F_3 + F_4$$

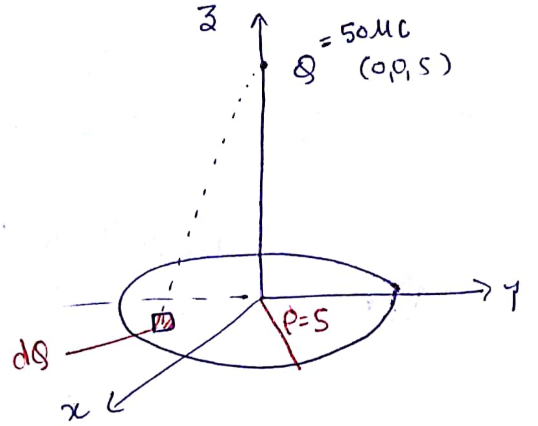
$$\vec{F} = \frac{Q}{4\pi\epsilon} \frac{4R\hat{a}_z}{(\sqrt{2}R)^3}$$

$$|\vec{F}| = \frac{Q}{2\sqrt{2}\pi\epsilon R^2} \text{ Newton}$$

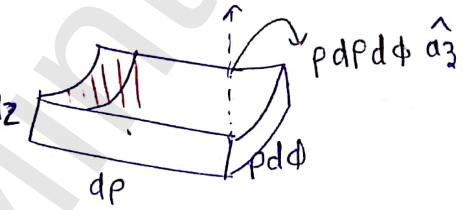
Hint

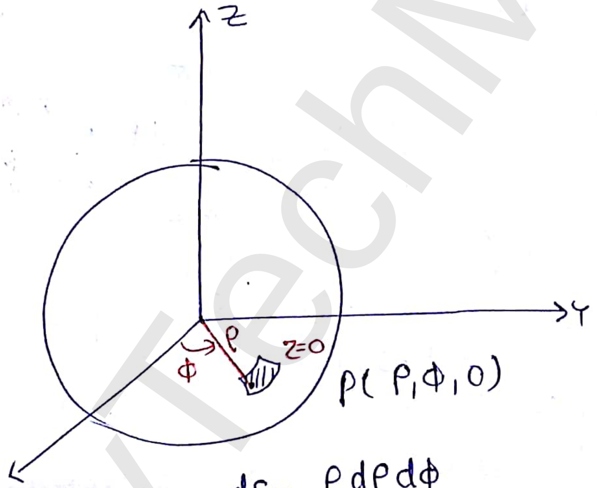
Q Find the Force on a point charge of 50μC at 00S meter due to a Charge of 500π μC i.e uniformly distributed over the circular disc $\rho \leq 5m$ and $z = 0m$

Soln



$$F = \frac{(dQ)\cdot Q}{4\pi\epsilon|\rho|^2}\ \vec{R}_{12}$$





$$ds = \rho\, d\rho\, d\phi$$

$$dQ = \rho_s\, ds$$

$$\vec{R}_{12} =$$

$$= \rho_s\, \rho\, d\rho\, d\phi$$

$$\rho_s = \frac{\text{Charge}}{\text{area}} = \frac{500\pi \times 10^{-6}}{\pi\, 5^2} \left(\frac{C}{m^2}\right)$$

$$\vec{R}_{12} = \vec{OP_2} - \vec{OP_1}$$

$$= (5-0)\hat{a}_z - [(\rho-0)]\,\hat{a}_\rho$$

$$\vec{R}_{12} = -\rho\hat{a}_\rho + 5\hat{a}_z$$

$$\vec{R}_{12} = \sqrt{\rho^2+5^2}$$

$$\vec{dF}_{12} = \frac{dQ_1\, Q_2\, \vec{R}_{12}}{4\pi\epsilon\, |\vec{R}_{12}|^3}$$

$$\vec{dF}_{12} = \frac{(\rho_s \rho\, d\rho\, d\phi)\, 50\times10^{-6}\, [-\rho\hat{a}_\rho + 5\hat{a}_z]}{4\pi\epsilon\, [\sqrt{\rho^2+5^2}]^3}$$

→ From the symmetry of the problem it is understood that only z component is present

→ total Force is

$$\vec{F}_{12} = \iint \frac{\rho_s(\rho\, d\rho\, d\phi)(50\times10^{-6})\; 5\hat{a}_z}{4\pi\epsilon_0\, (\rho^2+5^2)^{3/2}}$$

skip do this ∫ integration

$$\vec{F}_{12} = \frac{\rho_s\,(50\times10^{-6})}{4\pi\epsilon_0} \int_{\rho=0}^{5} \frac{1}{2}\, \frac{2\rho\, d\rho}{(\rho^2+5^2)^{3/2}} \int_{\phi=0}^{2\pi} d\phi\; \hat{a}_z$$

$$= \frac{\rho_s(50\times10^{-6})}{4\pi\epsilon_0} \left[\frac{1}{2}\, \frac{(\rho^2+5^2)^{-3/2+1}}{-\frac{3}{2}+1}\right]_{\rho=0}^{5} 2\pi\, \hat{a}_z$$

$$= (20\times10^{-6})(250\times10^{-6})(9\times10^{9})\left[-\left[\frac{1}{\sqrt{5^2+5^2}} - \frac{1}{\sqrt{5^2}}\right]\right] 2\pi\, \hat{a}_z$$

Force $= 16.56\, \hat{a}_z$ (Newton)

Force on unit charge called E.F intensity. if we put 1 C in place of 50 μC then Answer will be E.F

Electric field Intensity ($\vec{E}$) is Force vector on +1C charge (unit +ve charge)

→ electric field intensity due to at P_2 due to Q charge located at P_1 is

$$\vec{E}_{12} = \frac{Q\, \vec{a}_{12}}{4\pi\epsilon|\vec{R}_{12}|^2} = \frac{Q\, \vec{R}_{12}}{4\pi\epsilon|\vec{R}_{12}|^3}$$

$$\vec{R}_{12} = P_2 - P_1$$

$\vec{R}_{12}$ = Field loc^n - Source loc^n

hint:- $dQ = \rho_l\, dl$ in case of line charge

$dQ = \rho_s\, ds$ in case of surface charge

$dQ = \rho_v\, dv$ in case of volume charge

$$d\vec{E}_{12} = \frac{dQ\, \vec{R}_{12}}{4\pi\epsilon|\vec{R}_{12}|^3}$$

→ in case of line charge

$$\vec{E}_{12} = \int \frac{\rho_l\, dl\, \vec{R}_{12}}{4\pi\epsilon\, |\vec{R}_{12}|^3}$$

→ in case of surface charge

$$E_{12} = \iint \frac{\rho_s\, ds\, \vec{R}_{12}}{4\pi\epsilon\, |\vec{R}_{12}|^3}$$

→ in case of volume charge

$$\vec{E}_{12} = \iiint \frac{\rho_v\, dv\, \vec{R}_{12}}{4\pi\epsilon\, |\vec{R}_{12}|^3}$$

Q Find $\vec{E}$ at (1,1,1) due to four identical 3nC point charges located at

$P_1(1\,1\,0)$ $P_3(-1,-1\,0)$

$P_2(-1\,1\,0)$ $P_4(1\,-1\,0)$

Soln

$$\frac{Q}{4\pi\epsilon} = \frac{3\times10^{-9}}{}(9\times10^{9}) = 27$$

$$\vec{E}_{12} = \frac{Q}{4\pi\epsilon}\frac{\vec{R}_{12}}{|\vec{R}_{12}|^3}$$

$$E = E_1 + E_2 + E_3 + E_4$$

$$E_1 = 27\left[\frac{(1-1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z}{(\sqrt{1^2})^3}\right]$$

$$E_2 = +27\left[\frac{(1+1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z}{\left[\sqrt{2^2+1^2}\right]^3}\right]$$

$$E_3 = 27\left[\frac{(1+1)\hat{a}_x + (1+1)\hat{a}_y + 1\hat{a}_z}{(3)^3}\right]$$

$$E_4 = 27\left[\frac{(1-1)\hat{a}_x + (1+1)\hat{a}_y + (1-0)\hat{a}_z}{(\sqrt{5})^3}\right]$$

$$\vec{E} = 6.82\,\hat{a}_x + 6.82\,\hat{a}_y + 32.8\,\hat{a}_z \left(\frac{\text{volt}}{m}\right)$$

Q $Q_1 = 300\mu C$ is at $(1, -1, -3)$ experiences a force $F_1 = 8\hat{a_x} - 8\hat{a_y} + 4\hat{a_z}$ (Newton) due to Q_2 point charge located at $(3, -3, -2)$ Find Q_2

$$F = \frac{Q_1 Q_2}{4\pi\epsilon} \frac{\vec{R_{12}}}{|\vec{R_{12}}|^3}$$

$$= \frac{300\times10^{-6} \times Q_2 \{(1-3)\hat{a_x} + (-1+3)\hat{a_y} + (-3+2)\hat{a_z}\}}{4\pi\epsilon \left(\sqrt{4+4+1}\right)^3}$$

$$= \frac{300\times10^{-6} \times Q_2\, 9\times10^{9}}{27} (-2\hat{a_x} + 2\hat{a_y} - \hat{a_z})$$

$$8 = \frac{\overset{100}{\cancel{300}} \times 10^{-6} \times Q_2 \times \cancel{9} \times \overset{10^3}{\cancel{10^9}} \times -2}{\cancel{3}\ \cancel{27}}$$

$$8 = 100 \times Q_2 \times 10^3 \times -2$$

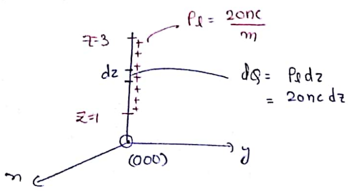
$$Q_2 = \frac{8 \times 10}{-2 \times 10^5 \times 10}$$

$$\boxed{Q_2 = -40\,\mu C}$$

Q A uniform line charge density of $20\frac{nC}{m}$ lies on the z axis from z=1 to 3 mtr

Doubts
① Find $\vec{E}$ at origin
② find $\vec{E}$ at (4 0 0)

Sol^n



① $E = \dfrac{Q}{4\pi\epsilon} \dfrac{\vec{R_{12}}}{|\vec{R_{12}}|^3}$

$\vec{R_{12}}$ = Force loc^n − source loc^n

$\vec{R_{12}} = (0-0)\hat{a_x} + (0-0)\hat{a_y} + (0-z)\hat{a_z}$

$\vec{dE} = \dfrac{dQ}{4\pi\epsilon} \dfrac{-z\hat{a_z}}{(\sqrt{z^2})^3}$

~~$\rho_l = 2\times 20\times 10^{-9}$~~

$$\int dE = \int_1^3 \frac{-20\times 10^{-9}\times 9\times 10^9 \cdot z}{z^3}\, dz\, \hat{a_z}$$

$$= -180 \int_1^3 \frac{1}{z^2}\, dz\, \hat{a_z}$$

$$= -180 \left.\frac{z^{-2+1}}{-2+1}\right]_1^3 \hat{a_z}$$

$$= 180\left(\frac{1}{3} - \frac{1}{1}\right) \hat{a_z}$$

$$= 180\left(1 - \frac{1}{3}\right)(-\hat{a_z})$$

$$= 180 \times \frac{2}{3} =$$

$$= 120\,(-\hat{a_z})\ \left(\frac{V}{m}\right)$$

② $$E = \int dE = \int_1^3 -20\times 10^{-9}\times 9\times 10^9 \cdot \frac{(4-0)\hat{a_x} + (0-0)\hat{a_y} + (0-z)\hat{a_z}}{(\sqrt{16+z^2})^3}$$

$$E = -180 \int_1^3 \frac{(4\hat{a_x} - z\hat{a_z})\, dz}{(\sqrt{16+z^2})^3}$$

$$E = -180 \int_1^3 \frac{4\, dz}{(\sqrt{16+z^2})^3}\hat{a_x} + 180 \int_1^3 \frac{z\, dz}{(\sqrt{16+z^2})^3}$$

put $z = 4\tan\theta$, $dz = 4\sec^2\theta\, d\theta$

$$= -180 \int_1^3 \frac{4\cdot 4\sec^2\theta\, d\theta}{(\sqrt{16+16\tan^2\theta})^3}$$

$$= \frac{-180\cdot 4^2}{4^3} \int_{14.04^\circ}^{36.86^\circ} \frac{\sec^2\theta}{\sec^3\theta}\, d\theta$$

$$= \frac{-180}{4} \int_{14.04^\circ}^{36.86^\circ} \cos\theta\, d\theta$$

$$= -\frac{180}{4}\left[\sin\theta\right]_{14.04}^{36.86}$$

$$= -\frac{180}{4}\left[\frac{z}{\sqrt{4^2+z^2}}\right]_1^3$$

$$= -\frac{180}{4}\left[\frac{3}{5} - \frac{1}{\sqrt{17}}\right]\hat{a_x}$$

$$+ 180 \int_1^3 \frac{1}{2} \frac{2z\, dz}{(16+z^2)^{3/2}}$$

$$+ 180\times\frac{1}{2}\left[\frac{(16+z^2)^{-3/2+1}}{-\frac{3}{2}+1}\right]_1^3$$

$$+ 90\left[\frac{(25)^{-1/2}}{-\frac{1}{2}} - \frac{(17)^{-1/2}}{-\frac{1}{2}}\right]$$

$$+ 90\left[\frac{\frac{1}{5}}{-\frac{1}{2}} - \frac{\frac{1}{\sqrt{17}}}{-\frac{1}{2}}\right]$$

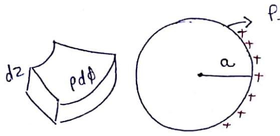
$$+ 90\left[-\frac{2}{5} + \frac{2}{\sqrt{17}}\right]$$

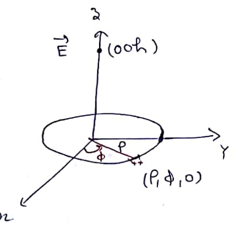
$$+ 7.6564\,\hat{a_z}$$

$$\boxed{= 16.09\,\hat{a_x} - 7.64\,\hat{a_z}}$$ Final Answer.

Q. A circular ring of radius 'a' carries uniform line charge density $\rho_l \left(\frac{C}{m}\right)$. Ring is on xy plane with axis coinciding with z axis. Find $\vec{E}$ at (0 0 h)

Sol^n





$$E = \frac{Q}{4\pi\epsilon} \frac{\vec{R_{12}}}{|\vec{R_{12}}|^3}$$

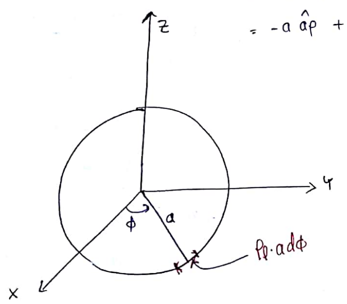
$\vec{R_{12}}$ = Field loc^n − Source loc^n

$= (0-a)\hat{a_\rho} + (\phi-\phi)\hat{a_\phi} + (h-0)\hat{a_z}$

$= -a\,\hat{a_\rho} + h\,\hat{a_z}$

$= -a\,\hat{a_\rho} + h\,\hat{a_z}$

By Sir.



$$dl = d\rho\,\hat{a_\rho} + \rho d\phi\,\hat{a_\phi} + dz\,\hat{a_z}$$

$$dE_{12} = \frac{dQ\,\vec{R_{12}}}{4\pi\epsilon|\vec{R_{12}}|^3}$$

$$d\vec{E_{12}} = \frac{\rho_l\,a\,d\phi\,[-a\,\hat{a_\rho} + h\,\hat{a_z}]}{4\pi\epsilon\,(\sqrt{a^2+h^2})^3}$$

→ From the symmetry of the problem only z component is present ($\hat{a_\rho}$ components cancel out)

total $\vec{E}$ is

$$\vec{E_{12}} = \frac{\rho_l \cdot a}{4\pi\epsilon}\int_{\phi=0}^{2\pi}\frac{h\,d\phi}{(a^2+h^2)^{3/2}}\,\hat{a_z} = \frac{\rho_l\,a\,h}{4\pi\epsilon\,(a^2+h^2)^{3/2}}\int_{\phi=0}^{2\pi} d\phi\,\hat{a_z}$$

$$\vec{E} = \frac{a h \rho_l}{2\epsilon\,(a^2+h^2)^{3/2}}\,\hat{a_z}$$

Gauss Law · states that the total electric flux crossing any closed surface is equals to the total charge inclosed by that closed surface.

$$\boxed{\oiint D \cdot ds = Q_{inclosed}}$$

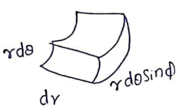
Q 25 μc point charge is located at origin · find flux passing through portion of the sphere $R = 20$ cm $\theta = 0$ to π, $\phi = 0$ to $\frac{\pi}{2}$

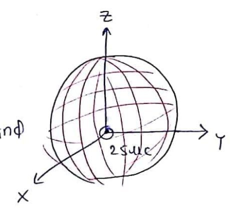
(ii) the surface $\rho = 0.8$m, $z = \pm 0.5$ m ~~the plane z=4 m~~

(iii) the plane $z = 4$ m.

Sol^n

$$\iint D \cdot ds$$





$$D = \frac{Q}{Area} = \frac{25 \times 10^{-6}}{4\pi(20 \times 10^{-2})^2}$$

$$= \int_{\phi=0}^{\pi/2}\int_{\theta=0}^{\pi} \frac{25 \times 10^{-6}}{4\pi(20 \times 10^{-2})^2} \times r\,d\theta \sin\phi$$

$$\frac{25 \times 10^{-6} \times 20 \times 10^{-2}}{4\pi \times 20 \times 10^{-4} \times 20}$$

By Sir

$0 \le \phi \le \pi/2$ means Range of ϕ is divided with 4, so total flux also divides with 4.

$$\psi\Big|\frac{(\text{closed Surface})}{4} = \frac{25}{4} = 6.25\ \mu C$$

(ii) $\rho = 0.8$m, $z = \pm 0.5$m is a closed surface

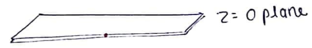
$$\psi|_{\text{closed surface}} = 25\ \mu C$$

(iii) flux crossing $z = 4$m surface $= \frac{25}{2}$ μc

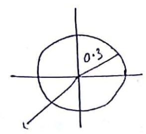
Important

Q 25μc point charge is at origin find flux crossing $z=0$ plane.

Sol^n 25μc



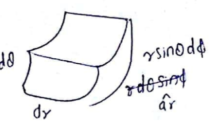
Q $\vec{D} = \frac{r}{3}\hat{a_r}$ nc/m^2 in free space Find total flux leaving the sphere $r = 0.3$ m



Sol^n $\oiint D \cdot ds$

$$= \oiint \frac{r}{3}\hat{a_r}\ r\,d\theta \sin\phi\ \hat{a_r}$$

$$\oiint_{\phi=0,\ \theta=0}^{2\pi,\ \pi} \frac{r^2}{3} d\theta\, d\phi \sin\phi$$



$$= \frac{(0.3)^2}{3}$$

$$Q = \iint \left(\frac{r}{3}\times 10^{-9}\right)\hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$\frac{(0.3)^3}{3}\times 10^{-9} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi_0}^{2\pi} d\phi$$

$$\frac{(0.3)^2}{3}\times 10^{-9} \; (2)\;(2\pi)$$

$= 0.113\, nC$

Q Find divergence of electric field due to Q point charge located at origin in all regions

$\nabla \cdot$

$\frac{\partial}{\partial x} x +$

(x, y, z)

$x\hat{i} + y\hat{j} + z\hat{k}$

(000)

By Sir·

$P(r, \theta, \phi)$

$E = \frac{Q}{4\pi\epsilon r^2}\hat{a}_r$

$\vec{E} = E_r \cdot \hat{a}_r$

$$\nabla \cdot E = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u} h_2 h_3 A_u\right]$$

$$\nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u} h_2 h_3 A_u + \frac{\partial}{\partial v} h_1 h_3 A_v + \frac{\partial}{\partial w} h_1 h_2 A_w\right]$$

$$= \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r} r^2 \sin\theta \cdot \frac{Q}{4\pi\epsilon r^2}\right]$$

$P(r, \theta, \phi)$

$\rho_v = \frac{Q}{\Delta V}$

$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r \quad \Big] \; r>0$

$\Delta v \to 0 \leftarrow$ dign of point

origin

$\vec{D} = D_r \hat{a}_r$

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \cdot \frac{Q}{4\pi r^2}$$

$\vec{\nabla} \cdot \vec{D} = 0$

at $r=0$ $\vec{D} = \infty$ so $\nabla \cdot \vec{D}$ don't exist at $r=0$

but actually $\vec{\nabla} \cdot \vec{D} = \rho_v$

so problem is that

actually at $r=0$ $\vec{\nabla} \cdot \vec{D}$ don't exist if exist then it is very large value and can't be determined.

$\vec{\nabla} \cdot \vec{D} \neq 0$ at $r=0$ $(\nabla \cdot \vec{D} = \rho_v)$

$\vec{\nabla} \cdot \vec{D} = 0$ for $r>0$

$\vec{\nabla} \cdot \vec{E} \neq 0$ at $r=0$ $\left[\vec{\nabla} \cdot \vec{E} = \frac{\rho_v}{\epsilon}\right]$

$\vec{\nabla} \cdot \vec{E} = 0$ for $r>0$

but this value is very high bcz ρ_v is very high bcz $\rho_v = \frac{Q}{\Delta v}$

Δv is very small

1) $\vec{D} = 20xy^2(z+1)\hat{a}_x + 20x^2y(z+1)\hat{a}_y + 10x^2y^2\hat{a}_z \left(\frac{C}{m^2}\right)$

Find $\nabla\cdot\vec{D}$ at $P(0.3, 0.4, 0.5)$

Solⁿ $\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

$\vec{\nabla}\cdot\vec{D} = 20y^2(z+1) + 20x^2(z+1) + 0$

$\vec{\nabla}\cdot\vec{D}\Big|_{(0.3,0.4,0.5)} = 20(0.4)^2(0.5+1) + 20(0.3)^2(0.5+1)$

$= 7.5 \ \frac{C}{m^3}$

Q $\vec{D} = 4\rho z\sin\phi\,\hat{a}_\rho + 2\rho z\cos\phi\,\hat{a}_\phi + 2\rho^2\sin\phi\,\hat{a}_z \left(\frac{C}{m^2}\right)$

Find $\nabla\cdot\vec{D}$ at $(1, \frac{\pi}{2}, 2)$

$\nabla\cdot\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\cdot D_\rho + \frac{1}{\rho}\frac{\partial}{\partial\phi}D_\phi + \frac{1}{\rho}\frac{\partial}{\partial z}\rho D_z$

$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\cdot 4\rho z\sin\phi + \frac{1}{\rho}\frac{\partial}{\partial\phi}\rho z\cos\phi + \frac{\partial}{\partial z}2\rho^2\sin\phi$

$= \frac{4z\sin\phi\,(2\rho)}{\rho} + \frac{1}{\rho}\,2\rho z(-\sin\phi) + 0$

$\vec{\nabla}\cdot\vec{D}\Big|_{(1,\frac{\pi}{2},2)} = 8(2)\sin 90^\circ + 2(2)(-\sin 90^\circ)$

$= 16 - 4$

$= 12 \ \frac{C}{m^3}$

Q $\rho_v = \frac{5\cos^2\phi}{r^4} \ \frac{C}{m^3}$ find the charge present in region $1 \le r \le 2m$

$\nabla\cdot D = \rho_v$

Note* r given means spherical coordinate but some time r given but still cylindrical coordinate

$\rho_v = \frac{5\cos^2\phi}{r^4} \ \frac{C}{m^3}$ find the charge present in cylindrical region $1 \le r \le 2m$. then treat r as ρ

$Q = \iiint \rho_v\, dv$

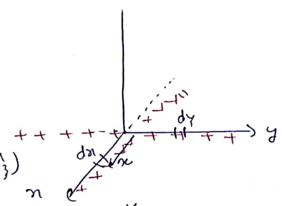
$= \iiint \left(\frac{5\cos^2\phi}{r^4}\right) r^2\sin\theta\, dr\, d\theta\, d\phi$

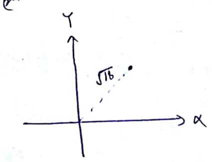
$= 5\int_{r=1}^{2}\frac{dr}{r^2}\int_{\theta=0}^{\pi}\sin\theta\, d\theta\int_{\phi=0}^{2\pi}\cos^2\phi\, d\phi = 5\pi$

Q Find two identical line charges lie along x and y axis with $\rho_L = \frac{20\mu C}{m}$ find $\vec{D}$ at $(3,3,3)$

$E = \frac{Q}{4\pi\epsilon|R_{12}|^3}R_{12}$

$\frac{20\times10^{-6}\,dx\,(3-x)\hat{a}_x + 3\hat{a}_y + 3\hat{a}_z)}{4\pi\epsilon\sqrt{(3-x)^2+3^2+3^2}}$

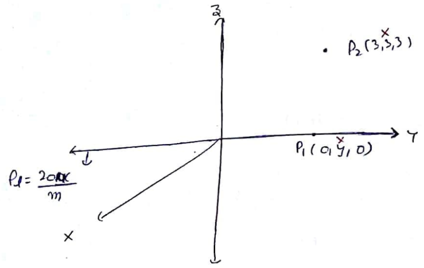




$$\vec{E} = \frac{\rho_l}{2\pi\epsilon\rho}\hat{a}_\rho \quad \{ \text{due to } \infty \text{ long line} \}$$

ρ is ⊥ distance from line charge to field location.

→ $\hat{a}_\rho$ is unit vector along ⊥r line drawn from line charge to field location.



Case 1 $\vec{D}$ at (3,3,3) due to x line

→ $\vec{D}$ is independent of x value

$P_2(3\,3\,3)$ $P_1(x,0,0)$

$\vec{R_{12}}$ = (field point) − source point

$\vec{R_{12}} = (3-0)\hat{a}_y + (3-0)\hat{a}_z$

$\vec{R_{12}} = 3\hat{a}_y + 3\hat{a}_z$

→ $\vec{R_{12}}$ is ⊥r vector from x axis (line charge) to field point

→ $\rho = |\vec{R_{12}}|$

$\rho = \sqrt{3^2+3^2}$

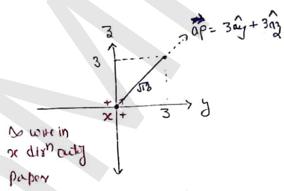
$$\hat{a}_\rho = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{3\hat{a}_y + 3\hat{a}_z}{\sqrt{3^2+3^2}}$$

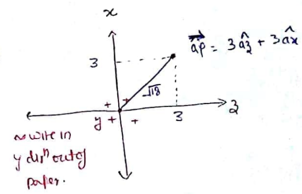
$$\vec{D_x} = \frac{\rho_l}{2\pi\sqrt{3^2+3^2}}\left(\frac{3\hat{a}_y + 3\hat{a}_z}{\sqrt{3^2+3^2}}\right)$$

Similarly

$$\vec{D_y} = \frac{\rho_l}{2\pi\sqrt{3^2+3^2}}\left(\frac{3\hat{a}_x + 3\hat{a}_z}{\sqrt{3^2+3^2}}\right)$$

total $\vec{D} = \vec{D_x} + \vec{D_y}$





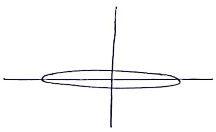
Q. $\vec{J} = \frac{4}{r^2}\cos\theta\,\hat{a}_r + 20e^{-2r}\sin\theta\,\hat{a}_\theta - r\sin\theta\cos\phi\,\hat{a}_\phi \quad \left(\frac{A}{m^2}\right)$

Find total current passing through the spherical cap. $r=3$;

$0 < \theta < \frac{\pi}{6} \quad , 0 < \phi < 2\pi$

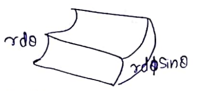
$J = \frac{I}{A}$

$I = J \cdot ds$



x component crosses x=const surface

→ $\hat{a}_r$ component crosses r = const (r=3)



$$I\Big|_{\substack{crossing \\ r=3\ surface}} = \iint \left(\frac{4}{r^2}\cos\theta\,\hat{a}_r\right) \cdot r^2\sin\theta\,d\theta\,d\phi$$

$$= 2\int_{\theta=0}^{\pi/6} 2\cos\theta\sin\theta\,d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= 2\left(-\frac{\cos 2\theta}{2}\right)_{0=0}^{\pi/6} (\phi)_{\phi=0}$$

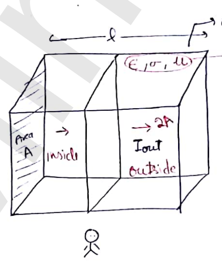
$$= -\left[\cos\frac{\pi}{3} - \cos 0\right](2\pi)$$

$$= -\left[\frac{1}{2} - 1\right](2\pi)$$

$$= \pi$$

Date- 24 sept 2017

Continuity equation :



→ these three paramtr are present in condc

$R = \frac{l}{\sigma A}$: $C = \frac{\epsilon A}{l}$

$RC = \frac{l\epsilon A}{\sigma A \cdot l}$

→ condition to have continuous current is

$$I_{out} = -\frac{dq_{in}}{dt} \rightarrow ①$$

continuity eqn says if there is a const current of 2A in outside region तो inside region में भी charge लगातार decrease करना चाहिए।

$$I = -\frac{dq}{dt} \quad — ①$$

→ by definition current crossing closed surface

$$I = \oint \vec{J} \cdot \vec{ds} \quad — ②$$

→ charge with in a volume is

$$q = \iiint \rho_v \cdot dv \quad — ③$$

② ③ in ①

$$\oint \vec{J} \cdot \vec{ds} = -\frac{d}{dt}\left(\iiint \rho_v \cdot dv\right) \quad — ④$$

by divergence theorem

$$\oint \vec{J} \cdot \vec{ds} = \iiint (\nabla \cdot \vec{J})\,dv \quad — ⑤$$

⑤ in ④

$$\iiint (\vec{\nabla}\cdot\vec{J})\,dv = \iiint \left(-\frac{\partial \rho_v}{\partial t}\right) dv \quad \text{—(6)}$$

Compare both sides

$$\boxed{\vec{\nabla}\cdot\vec{J} = \frac{-\partial \rho_v}{\partial t}} \quad \text{—(7)}$$

ρ_v can be a fun^n of distance (x, y, z) and 't' (time) also.

∴ changed $-\frac{d}{dt}\iiint \rho_v\, dv$ in

$\iiint \left(-\frac{\partial \rho_v}{\partial t}\right) dv$

$$\vec{J} = \sigma\vec{E} \quad \text{—(8)}$$

(8) in (7)

$$\vec{\nabla}\cdot\sigma\vec{E} = \frac{-\partial \rho_v}{\partial t}$$

For homogeneous medium σ is constant

$$\sigma\,\vec{\nabla}\cdot\vec{E} = \frac{-\partial \rho_v}{\partial t} \quad \text{—(9)}$$

$$D = \epsilon\vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\epsilon} \quad \text{—(10)}$$

$$\text{(10) in (9)} \Rightarrow \sigma\left(\nabla\cdot\frac{\vec{D}}{\epsilon}\right) = \frac{-\partial \rho_v}{\partial t} \quad \text{—(11)}$$

For homogeneous medium

$$\frac{\sigma}{\epsilon}\,\vec{\nabla}\cdot\vec{D} = \frac{-\partial \rho_v}{\partial t} \quad \text{—(12)}$$

maxwell eq^n is

$$\vec{\nabla}\cdot\vec{D} = \rho_v \quad \text{—(13)}$$

(13) in (12)

$$\frac{\sigma}{\epsilon}\rho_v = \frac{-\partial \rho_v}{\partial t}$$

$$\boxed{\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon}\rho_v = 0} \quad \text{—(14)} \quad \text{continuity eq}^n$$

$$\boxed{\frac{\partial J}{\partial t} + \frac{1}{\gamma}J = 0}$$

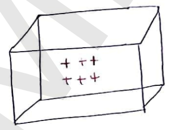
Let $\rho_0\left(\frac{C}{m^3}\right)$ is initial volume charge density at $t=0$ then

Solution of eq (14) is

$$\boxed{\rho_v(t) = \rho_0 \cdot e^{\frac{-t}{(\frac{\epsilon}{\sigma})}} = \rho_0\left(e^{-\frac{t}{\tau_r}}\right)}$$

or after solving continuity eq^n.

$$\boxed{\tau_r = \frac{\epsilon}{\sigma} = \text{Relaxation Time constant}}$$



If we leave a charge Q at $t=0$ after 5 time const it reaches at the surface of the conductor and cond^c become relaxed

at $t=0 \Rightarrow Q \quad \rho_0 = \frac{Q}{V}$

$$\tau = \frac{\epsilon}{\sigma} = \frac{8.85\times10^{-12}}{10^8}$$

$$5\tau = \frac{5\times8.85\times10^{-12}}{10^8}$$

$$\boxed{\tau = RC = \frac{\epsilon}{\sigma}}$$

Q1 Find magnitude of the Electric field intensity in a sample of silver having $\sigma = 6.17\times10^7\ \mho/m$ and $\mu_e = 0.0056\ \frac{m^2}{\text{Volt sec}}$

if
(a) drift velocity is $\frac{10\,mm}{sec}$
(b) current density is $10^7\ A/m^2$
(c) the sample is a cube of 3mm on a side carrying a total 8 amp
(d) The sample is a cube 3mm on a side having potential difference of 1mv b/w opposite faces.

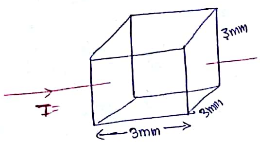
① $v_d = \mu E$

$$|\vec{E}| = \frac{v_d}{\mu} = \frac{10\,mm \times 10^{-3}\,m}{sec \times 56 \times 10^{-4}\,\frac{m^2}{sec}}$$

② $\vec{J} = \sigma \vec{E}$

$$|\vec{E}| = \frac{\vec{J}}{\sigma} = \frac{10^7}{6.17 \times 10^7}\ \frac{A}{m^2} \cdot \frac{m}{\mho}$$

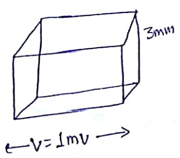
③



$$J = \frac{I}{Ar} = \frac{8\,A}{3 \times 3 \times 10^{-6}} = \frac{8}{9} \times 10^{6}\ \frac{A}{m^2}$$

$$\vec{E} = \frac{J}{\sigma} = \frac{\frac{8}{9} \times 10^{6}}{6.17 \times 10^{7}}$$

④



$$E = \frac{V}{d} = \frac{10^{-3}}{3 \times 10^{-3}} = \left(\frac{volt}{m}\right)$$

Note: Principles which applied to the conductors in static E.F are

① The static E.F intensity must be external to the cond^c and is everywhere directed normal to the cond^c surface.

② The conductor surface is an equipotential surface

③ The static E.F inside a conductor is zero.

① In dielectric Material $\boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P}}$ ← General Formula. — ①

we can use this Formula for non-linear material & linear material.

$\vec{P}$ is polarisation $\left(\frac{C}{m^2}\right)$

→ for linear dielectric material

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

$$\vec{P} = (\epsilon_r - 1)\epsilon_0 \vec{E} \rightarrow ②$$

$$\vec{D} = \epsilon \vec{E} \quad — ③$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\chi_e = \epsilon_r - 1 \Rightarrow \boxed{\epsilon_r = \chi_e + 1}$$

χ_e = electric susceptability (unit-less)

$$② \Rightarrow \vec{E} = \frac{\vec{P}}{(\epsilon_r - 1)\epsilon_0} \quad — ④$$

④ in ① $\vec{D} = \epsilon_0 \frac{\vec{P}}{(\epsilon_r-1)\epsilon_0} + \vec{P}$

$$\vec{D} = \vec{P}\left(\frac{1}{\epsilon_r-1}+1\right)$$

$$\vec{D} = \vec{P}\left[\frac{\epsilon_r}{\epsilon_r-1}\right]$$

$$\vec{P} = \left[\frac{\epsilon_r-1}{\epsilon_r}\right]\vec{D}$$

$$\boxed{\vec{P} = \left(1-\frac{1}{\epsilon_r}\right)\vec{D}}$$

Q a dielectric ~~also~~ slab has electric susceptibility 3 flux density is $3 \frac{C}{m^2}$ find polarization

$\epsilon_r = 1 + \chi_e$

$\epsilon_r = 1+3$

$\epsilon_r = 4$

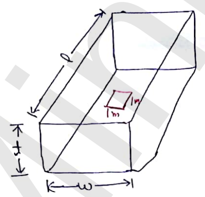
$\vec{D} = \frac{3C}{m^2}$

$P = \left(1-\frac{1}{\epsilon_r}\right) D$

$P = \left(1-\frac{1}{4}\right) 3\left[\frac{C}{m^2}\right]$

$$\boxed{\vec{P} = \frac{9}{4}\ \frac{C}{m^2}}$$

Observation :



R=

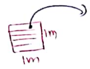
$R = \frac{\rho l}{A}$

$R = \frac{l}{\sigma(wt)}$ (Ω)

if $t \to 0$ then it is called as conductor surface [(or) sheet]

→ For conductor sheet resistance is calculated for a square area that $w = l$

$R_s = \frac{l}{\sigma l t} = \frac{1}{\sigma t}$.



$\frac{1}{\sigma t}$ is its resistance.

Note* Sheet [or surface] resistance (R_s) of a conducting surface having conductivity σ. and thickness 't' is

$$\boxed{R_s = \frac{1}{\sigma t}\ \left(\frac{\Omega}{square}\right)}$$

Q a conductive coating has 0.1 mm thickness having conductivity $3\times 10^6 \left(\frac{1}{\Omega \cdot m}\right)$ find sheet resistance.

Sol^n $R_s = \frac{1}{\sigma t} = \frac{1}{3\times10^6 (0.1\times10^{-3})}$ $\left(\frac{\Omega}{square}\right)$

Poisson equation :-

$$\vec{\nabla} \cdot \vec{D} = \rho_v \quad —① $$

$$\vec{D} = \epsilon \vec{E} \quad —②$$

② in ①

$$\nabla \cdot \epsilon \vec{E} = \rho_v \quad —③$$

$$\vec{E} = -\vec{\nabla} V \quad —④$$

④ in ③

$$\nabla \cdot [\epsilon(-\vec{\nabla} V)] = \rho_v$$

$$\boxed{\vec{\nabla} \cdot (\epsilon (\vec{\nabla} V)) = -\rho_v} \longrightarrow ⑤$$

this is a fun of x,y,z

inefective type . changes the posn of ϵ

Single crystal – non homogeneous material bcz dirn to dirn atomic arrangement changes.

eqn ⑤ is poisson equation equation of non homogeneous medium. for non homogeneous medium ϵ is a function of x, y, z.

→ For homogeneous medium

ϵ is constant so eqn ⑤ is

$$\cancel{\epsilon \nabla} \quad \epsilon \vec{\nabla} \cdot (\vec{\nabla} V) = -\rho_v \Rightarrow \boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \longrightarrow ⑥$$

eqn ⑥ is poisson equation for homogeneous eqn.

if $\rho_v = 0$ in eqn ⑤

(but Q, ρ_L, ρ_S can be present)

point charge, line charge, surface charge.

then $\vec{\nabla} \cdot [\epsilon (\vec{\nabla} V)] = 0$ is called as Laplace eqn for non homogeneous eqn.

→ if $\rho_v = 0$ in eq then $\boxed{\nabla^2 V = 0}$ is called as Laplace eqn for homogeneous ~~material~~ . medium .

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

↓ potential V can be find

$V(x,y,z)$ and two const C_1 and C_2 — 2 const bcz 2nd order D.E.

to find C_1 and C_2 two potential is given at two points.

if ρ_v given use laplace eqn to fin $C_{ab} = \frac{Q}{V_{ab}}$

$$C_{ab} = \frac{Q}{V_{ab}} = \frac{\psi}{V_{ab}} = \frac{\iint \vec{D} \cdot ds}{V_a - V_b}$$

V given at two point

$$\boxed{C_{ab} = \frac{Q}{V_{ab}} = \frac{\psi}{V_{ab}} = \frac{\iint \vec{D} \cdot ds}{-\int_b^a \vec{E} \cdot \vec{dl}}}$$

if ρ_v not given use poisson eqn to find V

If V found then can find E also

Uniqueness theorem : any solution of laplace, poisson eqn that satisfies the same boundary condn {initial voltage values} must be the only solution regardless of the method {analytical, graphical, numerical, experimental etc}, used

{two potentials given at two points i.e those are initial values or boundary condn}.

if we solve eqn by either laplace or poisson using same boundary condn the answer will be unique called uniqueness theorem.

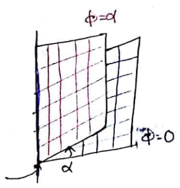
(Q) equipotential surfaces are given below

$V=0$ at $\phi=0$

$V=V_0$ at $\phi=\alpha$

Given $\rho_v=0$, homogeneous medium b/w the planes.

Find $V, \vec{E}$ equations b/w the planes



if let $\alpha=30°$ so he can ask. Potential at $V=V_0$ at $\phi=30°$.

Insulating gap. else two conducting planes will touch ~~so no potential difference~~ so same potential.

Soln if $\rho_v=0 \Rightarrow$ Laplace equation should be used

$\nabla^2 V=0$

→ in cylindrical $\nabla^2 V=0$

$$\nabla^2 V = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}\frac{h_2h_3}{h_1}\frac{\partial V}{\partial u} + \frac{\partial}{\partial v}\frac{h_1h_3}{h_2}\frac{\partial V}{\partial v} + \frac{\partial}{\partial w}\frac{h_1h_2}{h_3}\frac{\partial V}{\partial w}\right]$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

→ From given boundary condn V is a function of ϕ

So $\frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} = 0$ —— ①

$\frac{\partial^2 V}{\partial\phi^2} = 0$

Take integral

$$\int\frac{\partial}{\partial\phi}\left(\frac{\partial V}{\partial\phi}\right)\partial\phi = \int 0\,\partial\phi$$

$\frac{\partial V}{\partial\phi} = A$

$\partial V = A\,\partial\phi$

take integral

$\int dV = A\int\partial\phi$

$V = A\phi + B$ —— ②

$V=0$ at $\phi=0$

$0 = 0 + B \Rightarrow B=0$

$V = A\phi$

given $V=V_0$ at $\phi=\alpha$

$V_0 = A\alpha$

$A = \frac{V_0}{\alpha}$

$V = \frac{V_0}{\alpha}\phi$ —— ③

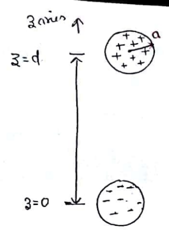
If can be asked let $\alpha=30°$, $V=4$ Volt

$\vec{E} = -\vec{\nabla}V$

$= -\left[\frac{\partial V}{\partial\rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$

$\vec{E} = -\left[\frac{1}{\rho}\frac{\partial}{\partial\phi}\left(\frac{V_0}{\alpha}\phi\right)\hat{a}_\phi\right] = -\frac{V_0}{\alpha\rho}\hat{a}_\phi$

Q



given $V = V_0$ at $z = d$
$V = 0$ at $z = 0$

Find V and E eqⁿ b/w plates.
Find capacitance b/w plates.

Solⁿ Nothing told about ϵ so air (homogeneous medium).

at $z=0$ potential = const
at $z=d$ " = "

in b/w distance changes and potential also changes.

Use Laplace eqⁿ $\nabla^2 V = 0$

(∵ ρ_s is source b/z charge present at surface) we said before that ρ_v may be 0 but point charge, surface charge may present.

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial V}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \underbrace{\frac{\partial^2 V}{\partial z^2}}_{\text{take this b/z z varying}} = 0.$$

From given boundary condⁿ V is a funᶜ of z

$$\frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial V}{\partial z} = A$$

$$V = Az + B \quad -①$$

$V = 0$ at $z = 0$

$0 = A\cdot 0 + B$

$\boxed{B=0}$

$V = Az \quad -②$

at $z=d$ $V=V_0$

$V_0 = A\cdot d$

$\boxed{A = \frac{V_0}{d}}$

$\rightarrow V = \cancel{A}\,\frac{V_0}{d}\cdot z \quad -③$

How V changes and changes in ‖ plate capᶜ.
can ask
$z=4$ $V = 10V$ Find V at $z=1m$ $V = \frac{V_0}{d}z = \frac{10}{4}(1)$
$z=0$ $\rightarrow V = 0V$

$\vec{E} = -\vec{\nabla} V$

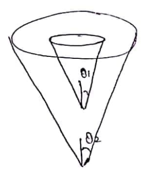
$\vec{E} = -\frac{\partial V}{\partial z}\hat{a}_z$

$\vec{E} = -\frac{\partial}{\partial z}\left(\frac{V_0}{d}z\right)a_z$

$\vec{E} = -\frac{V_0}{d}\hat{a}_z$

take ϵ only b/z homo medium is given.

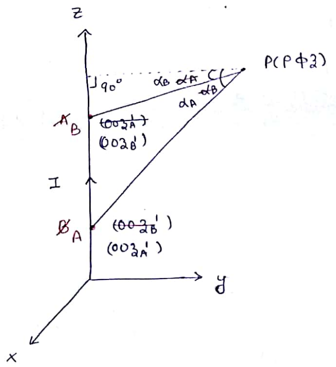
$$\rightarrow C_{+-} = \frac{|Q|}{V_{+-}} = \frac{|\Psi|}{V_{+-}} = \frac{\left|\iint \vec{D}\cdot d\vec{s}\right|}{(V_+)-(V_-)} = \frac{\left(\frac{\epsilon_0 V_0}{d}\right)(\text{area})}{V_0 - 0} = \frac{\epsilon_0}{d}\pi a^2$$



Find capᶜ of cone.

Note*

Note*:



$\vec{H}(\rho\phi z) = \frac{I}{4\pi\rho}\left[\sin\alpha_A - \sin\alpha_B\right] \hat{a}_H$

$\hat{a}_H = \hat{a}_l \times \hat{a}_\perp$

$\hat{a}_H$ is ~~uniform~~ unit vector in $\vec{H}$ direction.

$\hat{a}_l$ is " " in I direction.

$\hat{a}_\perp$ is unit " along $\perp$r line drawn from current line to field.

→ ρ is $\perp$r distance from current line to field point

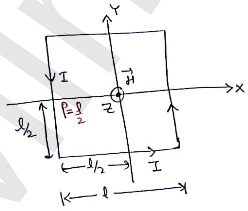
→ $\tan\alpha_A = \frac{z - z_A'}{\rho}$

α_A is angle b/w $\perp$r line and line drawn from field point to lower distance point.

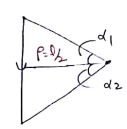
→ $\tan\alpha_B = \frac{z - z_B'}{\rho}$

α_B is angle b/w $\perp$r line and line drawn from field point to higher distance point

Q) Find $\vec{H}$ at mid location of square loop shown



Sol^n



$= \frac{I}{4\pi\rho}\left(\sin\alpha_1 + \sin\alpha_2\right) = \frac{I \cdot 2}{4\pi l} \cdot \left(\frac{2}{\sqrt{2}}\right) = \frac{I}{\sqrt{2}\pi l}$

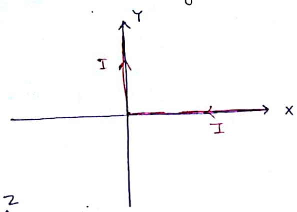
$\boxed{H_{Total} = \frac{4 \times I}{\sqrt{2}\pi l}\hat{a}_z = \frac{2\sqrt{2} I}{\pi l}\hat{a}_z}$

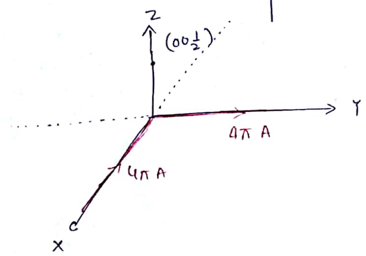
$B = \mu H$

$\vec{B} = \frac{\mu 2\sqrt{2} I}{\pi l}\hat{a}_z$

Q. A current of 4π amp. is directed from ∞ to origin on the x axis and back to ∞ along the +ve y axis. Find $\vec{H}$ at $(0\,0\,\frac{1}{2})$

Sol^n





we have two source. $H = H_x + H_y$

$$H_x = \frac{I}{4\pi\rho}\left[\sin\alpha_1 + \sin\alpha_2\right] = \frac{4\pi}{4\pi\cdot\frac{1}{2}}\left[\sin 90^\circ + \sin 0\right]$$

$$= 2\,\hat{a}_y$$

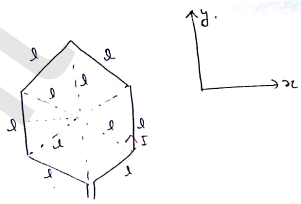
$$H_y = \frac{I}{4\pi\rho}\left[\sin 0 + \sin 90^\circ\right] = \frac{4\pi}{4\pi\cdot\frac{1}{2}}[1]$$

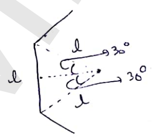
$$= 2\,\hat{a}_x$$

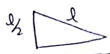
$$H_T = 2(\hat{a}_x + \hat{a}_y)$$

Q. Find the $\vec{H}$ at the mid loc^n of the Hexagon having side of "l"

Sol^n







$$\sqrt{l^2 - \frac{l^2}{4}} = \frac{\sqrt{3}}{2}l = \rho$$

$$= \frac{I}{4\pi\rho}\left[\sin 30^\circ + \sin 30^\circ\right]$$

bcz 6 cond^c

$$H_T = \frac{I\,2}{4\pi\sqrt{3}\cdot l}\left[\frac{2}{\cancel{2}}\right] \times 6 \quad \hat{a}_z$$

$B = \mu H = Wb/m^2 = $ Tesla

$$\vec{H} = \frac{I}{2\pi\sqrt{3}l} \times 6\ \hat{a}_z$$

$$\boxed{\vec{H}_T = \frac{\sqrt{3}I}{\pi l}\,\hat{a}_z}$$

No medium than take air (μ_0).

Ampere's Circuital Law :- Says closed line integral of magnetic field intensity is equal to the total current enclosed by that closed line

$$\oint H \cdot dl = I_{inclosed}$$

cause = strength x length

$$I = H \times \oint dl$$

$$\text{Curl} = \frac{\text{Circulation}}{\text{area}} = \frac{\int H \cdot dl}{ds} = \frac{I}{ds}$$

$$\nabla \times \vec{H} = \vec{J}$$

Q A Solid non magnetic cond^c of circular cross-section has its axis on the z axis, radius 4 mm and carries a uniformly distributed current of 50A. in a_z dir^n.

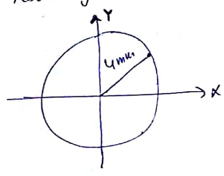
(a) find B at $\rho = 5$ mm

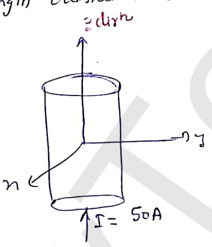
(b) find B at $\rho = 3$ mm

(c) find total magnetic flux per unit length inside the cond^c

(d) find total magnetic flux per unit length outside the cond^c.

Sol^n





(a)

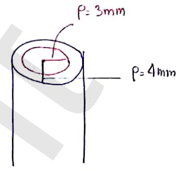
$$\oint H \cdot dl = I$$

$$H \cdot 2\pi 5 = 50$$

$$H = \frac{50}{2\pi 5 \times 10^{-3}} = \frac{50000}{2\pi 5} = \frac{10000}{2\pi} \hat{a_\phi}$$

$$B = \mu H = \frac{10000 \cdot 4\pi \times 10^{-7}}{2\pi} = 20 \times 10^{-4} \ (\text{Tesla} = Wb/m^2)$$

(b) $\rho = 3$ mm



$$\oint H \cdot dl = I_{inclosed}$$

$$H \cdot 2\pi \cdot 3 = \frac{I}{\pi(4)^2} \times (3)^2 \pi$$

$$H \cdot 2\pi 3 = \frac{I \times 9}{16}$$

$$H = \frac{9I}{16 \times 2\pi 3} \hat{a_\phi}$$

$$H = \frac{9 \times 50 \times}{16 \times 2 \times \pi \times 3 \times 10^{-3}} \hat{a_\phi} =$$

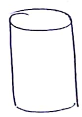
$$B = \frac{9 \times 50 \times 10^{3}}{16 \times 2 \times \pi \times 3} \times 4\pi \times 10^{-7}$$

$$= \frac{75}{4} \times 10^{-4}$$

$$B = 18.75 \times 10^{-4} \ Wb/m^2 \cdot \text{Tes} \ \hat{a_\phi}$$

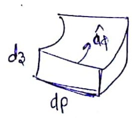
(c) given $\vec{B}$ then magnetic flux

$$\boxed{\Phi = \vec{B} \times \vec{A}}$$ ← dont use this here. use when B is constt



$$\Phi = \iint B \cdot ds$$

$$= \iint \frac{\mu_0 I \rho}{2\pi a^2} \hat{a_\phi} \cdot (d\rho \, dz) \hat{a_\phi}$$



$$= \frac{\mu_0 I}{2\pi a^2} \int_{\rho=0}^{a} \rho \, d\rho \int_{z=0}^{h} dz$$

$$\Phi = \frac{\mu_0 I}{2\pi a^2}\left(\frac{a^2}{2}\right) h$$

$$\frac{\Phi}{h} = \frac{\mu_0 I}{2\pi}$$

$$\frac{4\pi \times 10^{-7} \times 50}{2\pi\,(2)}$$

$$= 5 \left(\frac{\mu Wb}{m}\right)$$

(d) $\vec{B} = \frac{\mu_0 I}{2\pi\rho}\,\hat{a}_\phi$ → Formula for outside.

$$\Phi = \iint \vec{B}\cdot\vec{ds}$$

$$\Phi = \iint \frac{\mu_0 I}{2\pi\rho}\,\hat{a}_\phi \cdot d\rho\, dz\, \hat{a}_\phi$$

$$\Phi = \frac{\mu_0 I}{2\pi}\int_{\rho=a}^{\infty}\frac{d\rho}{\rho}\int_{z=0}^{h} dz$$

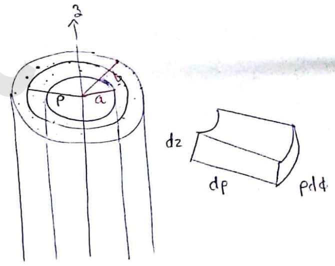
$$\frac{\Phi}{h} = \frac{\mu_0 I}{2\pi}\left[\ln(\rho)\right]_{\rho=a}^{\infty} \Rightarrow \begin{cases} \frac{\Phi}{h} = \frac{\mu_0 I}{2\pi}(\ln\infty - \ln a) \\ \frac{\Phi}{h} = \infty \end{cases}$$

Q $\vec{J}(\rho,\phi,z) = \begin{cases} 0 & \text{for } 0<\rho<a \\ \frac{2\rho}{a^2}\,\hat{a}_z & \text{for } a<\rho<b \end{cases}$

→ find $\vec{H}$ in the region $a<\rho<b$.

Soln

Soln



$$\vec{J} = \frac{2\rho}{a^2}\,\hat{a}_z$$

$$\oint H\cdot dl = I_{inc}$$

$$H_\phi 2\pi\rho = \iint \vec{J}\cdot ds$$

$$H_\phi 2\pi\rho = \iint\left(\frac{2\rho}{a^2}\,\hat{a}_z\right)\cdot(\rho\, d\rho\, d\phi)\,\hat{a}_z$$

$$H_\phi 2\pi\rho = \int_{\rho=a}^{\rho}\int_{0}^{2\pi}\frac{2\rho^2}{a^2}\,d\rho\, d\phi$$

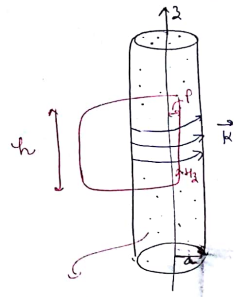
$$H_\phi 2\pi\rho = \frac{2\pi\cdot 2}{a^2}\left[\frac{\rho^3}{3}\right]_a^{\rho}$$

$$H_\phi 2\pi\rho = \frac{2\pi\cdot 2\cdot 1}{a^2\cdot 3}\left[\rho^3 - a^3\right]$$

$$H_\phi = \frac{2\pi\cdot 2}{2\pi\rho\cdot a^2\cdot 3}\left[\rho^3 - a^3\right] \ \text{A/m} \qquad \vec{H} = H_\phi\,\hat{a}_\phi$$

Q An ideal solenoid shown below has circular current sheet having $\vec{K} = K_0 \hat{a}_\phi$ on $\rho = a$ radius as shown. Find $\vec{H}$ inside solenoid.

Soln



$\vec{K} = K_0 \hat{a}_\phi$

magnetic material so No current inside.

$\vec{H}$ in the region $0 < \rho < a$

$$\oint \vec{H} \cdot \vec{dl} = I_{inc}$$

$$H_z \cdot h = K_0 \cdot h$$

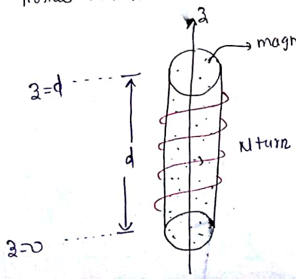
$$H_z = K_0$$

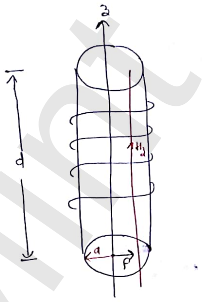
$$\vec{H} = H_z \hat{a}_z$$

$$\boxed{\vec{H} = K_0 \hat{a}_z \text{ for } 0 < \rho < a \quad \left(\tfrac{A}{m}\right) \left(\tfrac{A}{m}\right)}$$

$\vec{H} = 0$ outside $a < \rho < \infty$

Q Find $\vec{H}$ inside N turn solenoid shown





$\vec{H}$ in the region $0 < \rho < a$

$$\oint \vec{H} \cdot \vec{dl} = I_{incl}$$

$$H_z d = NI$$

$$H_z = \frac{NI}{d}$$

$$\vec{H} = H_z \hat{a}_z$$

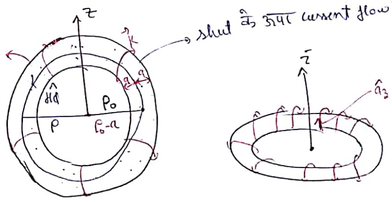
$$\boxed{\vec{H} = \frac{NI}{d} \hat{a}_z \text{ for } 0 < \rho < a}$$

$\vec{H} = 0$ for $a < \rho < \infty$

Q Find $\vec{H}$ inside ideal toroid shown below having $\vec{K} = K_0 \hat{a}_z \left(\tfrac{A}{m}\right)$ at $\rho = \rho_0 - a$, $z = 0$

magnetic material so No current.

→ sheet के ऊपर current flow हो रहा है



→ ① $\vec{H}$ in the region.

$0 < \rho < a$

$$\oint \vec{H} \cdot \vec{dl} = I_{incl}$$

$$H_\phi \, 2\pi\rho = K_0 \, 2\pi (\rho_0 - a)$$

$$H_\phi = \frac{K_0 (\rho_0 - a)}{\rho}$$

$$\vec{H} = H_\phi \hat{a}_\phi$$

$(\rho_0 - a) < \rho < \rho_0 + a$

$$\boxed{\vec{H} = K_0 \left(\frac{\rho_0 - a}{\rho}\right) \hat{a}_\phi \text{ for } \cancel{0 < \rho < a} \text{ inside toroid}}$$

$\vec{H} = 0$ for $\cancel{a < \rho < \infty}$ $\rho < a < \infty$ outside the toroid.

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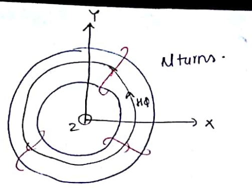
Note*



$\vec{H}$ at P radius

$\oint \vec{H} \cdot \vec{dl} = I_{inc}$

$H_\phi \, 2\pi\rho = NI$

$H_\phi = \frac{NI}{2\pi\rho}$

$\vec{H} = H_\phi \, \hat{a}_\phi$

$$\boxed{\begin{array}{l} \vec{H} = \frac{NI}{2\pi\rho} \hat{a}_\phi \text{ inside the toroid} \\ \vec{H} = 0 \text{ outside the toroid.} \end{array}}$$

Q In a certain region $\vec{B} = 5 \times 10^{-4} \, \hat{a}_z$ Tesla and

$\vec{E} = 5 \hat{a}_z \left(\frac{\text{volt}}{m}\right)$

a proton $\left\{ \begin{array}{l} q = 1.6 \times 10^{-19} \, C \\ m = 1.673 \times 10^{-27} \, kg \end{array} \right\}$ enters the field at the origin

with an initial velocity $v_0 = 2.5 \times 10^{5} \, \hat{a}_x \left(\frac{m}{sec}\right)$

→ what is the path of the proton

$F = qE + q(\vec{v} \times \vec{B})$

$= q5\hat{a}_z + q(2.5 \times 10^{5} \hat{a}_x \times 5 \times 10^{-4} \hat{a}_z)$

velocity of proton is in the dirn of of Force,

so due to $\vec{E}$ velocity of proton is $\hat{a}_z$.

→ due to $\vec{B}$ force on proton is circular in clockwise.

So proton path is helix path with axis as z axis

→ hint

$\vec{F}_{\text{magnetic field}} = q(\vec{v} \times \vec{B})$

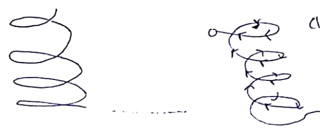
$= q(\hat{a}_x \times \hat{a}_z) = -\hat{a}_y$

$= q(-\hat{a}_y \times \hat{a}_z) = -\hat{a}_x$

$= q(-\hat{a}_x \times \hat{a}_z) = \hat{a}_y$

$= q(\hat{a}_y \times \hat{a}_z) = \hat{a}_x$

If no e.f then only circular motion (clockwise)



Clockwise helix with z axis.

Q Find force on a straight conductor of length 4 m carrying 2A current in $\hat{a}_z$ direction in the presence of field

$\vec{B} = 3 \times 10^{-4} (-a_x + a_y)$ Tesla   x y z

$\vec{F} = I \, \vec{l} \times \vec{B}$

$= 2\left[4\hat{a}_z \times 3 \times 10^{-4} (-\hat{a}_x + \hat{a}_y)\right]$

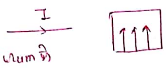
$= 24 \times 10^{-4} (-\hat{a}_y) + 24 \times 10^{-4} (-\hat{a}_x)$

$= 2.4 (-\hat{a}_y - \hat{a}_x)$ m (Newton)

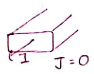
Note*:-

① $\vec{E} = -\vec{\nabla} V = -\text{grad } V$

$V =$ Scalar electric potential (volt)



② $\boxed{\underset{amp/m}{\vec{H}} = -\underset{1/m}{\vec{\nabla}} \underset{amp}{V_m}}$ $\boxed{\text{Provided } \vec{J} = 0}$ ← [illegible]



V_m = Scalar magnetic potential (ampere)

Checking: Maxwell eqn $\vec{\nabla} \times \vec{H} = \vec{J} \longrightarrow ③$

put ② in ③ $\quad \vec{\nabla} \times (-\vec{\nabla} V_m) = \vec{J}$

$-\vec{\nabla} \times \vec{\nabla} V_m = \vec{J}$

$0 = \vec{J}$

(iii) $\vec{\nabla} \vec{B} = 0 \longrightarrow ④$ (EMT)

$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0 \longrightarrow ⑤$ (maths)

$\vec{A}$ ↳ any vector

compare ④ ⑤

$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$

$\frac{wb}{m^2} = \cancel{\vec{\nabla}} \times \vec{A}$

$\quad\quad \frac{1}{m}$ —

$\vec{A}$ is called vector magnetic potential $\left(\frac{weber}{m}\right)$

$\vec{B} = \vec{\nabla} \times \vec{A}$

$\mu \vec{H} = \nabla \times \vec{A}$

$\boxed{\vec{H} = \frac{\nabla \times \vec{A}}{\mu}}$ — it has ability to produce $\vec{H}$ so called vector magnetic potential.

$\vec{B}_{12} = \mu \vec{H}_{12}$

$\underset{(wb/m^2)}{B_{12}} = \vec{\nabla} \times \underset{(wb/m)}{\vec{A}_{12}} = \int \frac{\mu I_1 \vec{dl_1} \times \vec{a}_{12}}{4\pi |\vec{R}_{12}|^2}$

$\underset{(wb/m)}{\vec{A}_{12}} = \int \frac{\mu I_1 \vec{dl_1}}{4\pi |\vec{R}_{12}|^2}$

Note*

① $\vec{A}_{12} = \int \frac{\mu I_1 \vec{dl_1}}{4\pi |\vec{R}_{12}|}$ $\left(\frac{weber}{m}\right)$

$\vec{A}_{12}$ is vector magnetic potential at P_2 due to I_1 current at P_1.

② $\frac{1}{2} \vec{A} \cdot \vec{J}$ = magnetic energy density

$\left(\frac{wb}{m}\right) \cdot \left(\frac{Amp}{m^2}\right) = \left(\frac{Joule}{m^3}\right)$

③ $\oint \vec{A} \cdot \vec{dl} = \iint (\vec{\nabla} \times \vec{A}) \vec{ds}$

$\underbrace{\frac{wb}{m} \cdot m}_{} = \underbrace{\left(\frac{1}{m}\right) \cdot \frac{wb}{m} \; m^2}_{}$

magnetic flux inside a closed line

magnetic flux crossing the open surface. (this surface is inside a closed line.)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{A}) = \vec{\nabla} \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\frac{1}{\mu}\left(\vec{\nabla} \times (\nabla \times \vec{A})\right) = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\frac{1}{\mu}\left[-(\vec{\nabla} \cdot \vec{\nabla} A)\right.$$

<u>observation</u> :-

① $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ — ①

$\vec{B} = \vec{\nabla} \times \vec{A}$ — ②

② in ① $\Rightarrow$ $\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{A})$

$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$

compare $\boxed{\vec{E} = -\frac{\partial \vec{A}}{\partial t}} \rightarrow$ ③

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$ — ④

$\vec{H} = \frac{1}{\mu}(\vec{\nabla} \times \vec{A})$ — ⑤

⑤ in ④.

$$\vec{\nabla} \times \frac{1}{\mu}(\vec{\nabla} \times \vec{A}) = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

For homogeneous medium

$$\frac{1}{\mu} \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$\frac{1}{\mu}\left[(\vec{\nabla} \cdot \vec{A})(\vec{\nabla}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{A}\right] = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$ — ⑥

$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{\nabla} \cdot \vec{A} = 0$ — ⑦

③ ⑦ in ⑥ $\Rightarrow$

$$\frac{1}{\mu}(-\nabla^2 \vec{A}) = \vec{J} + \epsilon \frac{\partial}{\partial t}\left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\boxed{\nabla^2 A - \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J}}$$

this is called poisson equation of magnetic fields (or) vector poisson equation for time varying fields.

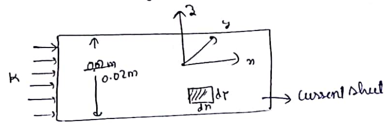
For static fields vector poisson eq^n is $\nabla^2 \vec{A} = -\mu \vec{J}$

if $\vec{J} = 0$ then

$\nabla^2 \vec{A} = 0$ is called laplace eq^n of magnetic field (or) vector laplace eq^n for static field

Q A current strip of 2 cm width carries 15 amp current in $\hat{a}_x$ direction as shown in figure.

→ Find force on current strip per unit length if uniform magnetic field having $\vec{B} = 0.2\,\hat{a}_y$ (Tesla)



hint :	$I\vec{dl}$	$=$	$\vec{K}\,ds$	$=$	$\bar{J}\,dv$
	A·m		$\left(\frac{A}{m}\right)m^2$		$\left(\frac{A}{m^2}\right)m^3$

Soln

$$\vec{dF} = I\vec{dl} \times \vec{B}$$

$$= \vec{K}\,ds \times \vec{B}$$

$$= \left(\frac{15}{0.02}\hat{a}_x\right)(dx\,dy) \times (0.2\hat{a}_y)$$

$$\vec{dF} = \frac{15}{0.02}(0.2)\,dx\,dy\,\hat{a}_z$$

total force on conductor surface is

$$\vec{F} = \frac{15}{0.02}(0.2)\int_{x=0}^{L} dx \int_{y=-0.01}^{+0.01} dy \;\hat{a}_z$$

$$= \frac{15\,(0.2)\,(L)\,(0.02)\,\hat{a}_z}{(0.02)}$$

$$\frac{\vec{F}}{L} = 3\,\hat{a}_z \;\frac{\text{Newton}}{m}$$

① Faradays law

$$\text{emf} = -\frac{d(N\Phi)}{dt}$$

$$\text{emf} = \frac{-N\,d\Phi}{dt} \text{ (volt)}$$

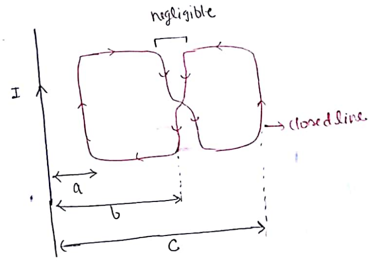
Q given magnetic flux is $3t^3 - 2t^2$ (where) find emf in a coil having 100 turns at $t = 6$ sec

$$e = -100\,\frac{d}{dt}(3t^3 - 2t^2)$$

$$e = -100\,(9t^2 - 4t)$$

$$e|_{t=6} = -100\left[9\cdot(6)^2 - 24\right]$$

Q Given total magnetic flux inside a closed line is 0. Find relation b/w a, b, c



given magnetic flux crossing the open surface present inside the closed line is zero

$$\iint \vec{B}\cdot\vec{ds} = 0$$

$$\iint \left(\frac{\mu I}{2\pi \rho}\hat{a}_\phi\right)\cdot (d\rho\, dz)\hat{a}_\phi = 0$$

$$\frac{\mu I}{2\pi}\int \frac{d\rho}{\rho}\int dz = 0$$

$$\left[\int_{\rho=b}^{a}\frac{d\rho}{\rho} + \int_{\rho=b}^{c}\frac{d\rho}{\rho}\right]\int_{z=0}^{h} dz = 0$$

$$(\ln a - \ln b + \ln c - \ln b)(h) = 0$$

$$\ln a + \ln c = 2\ln b$$

$$\ln ac = \ln b^2$$

$$\boxed{b^2 = ac}$$

Boundary conditions

$$\boxed{\vec{E}_{t1} = \vec{E}_{t2}}$$

$$\boxed{D_{N1} - D_{N2} = \rho_s}$$

$\left(\frac{C}{m^2}\right)$ $\left(\frac{C}{m^2}\right)$ $\frac{C}{m^2}$

$$\vec{H}_{t1} - \vec{H}_{t2} = \vec{a}_{N12} \times K$$

(A/m) (A/m) normal (A/m)

$$\boxed{\vec{B}_{N1} = \vec{B}_{N2}}$$

$\oint \vec{E}\cdot d\vec{l} = 0$ | $\oint\!\!\oint \vec{D}\cdot d\vec{s} = Q$

$\oint \vec{H}\cdot d\vec{l} = J$ | $\oint\!\!\oint \vec{B}\cdot d\vec{s} = 0$

① electric boundary condition

$$\boxed{\vec{E}_{t1} = \vec{E}_{t2}} \Rightarrow \frac{\vec{D}_{t1}}{\epsilon_1} = \frac{\vec{D}_{t2}}{\epsilon_2}$$

tangential compo. of E.F intensities are continious (equal) at the boundary surface.

$$\rightarrow \boxed{|\vec{D}_{N1}| - |\vec{D}_{N2}| = \rho_s}$$

$\frac{C}{m^2}$ $\frac{C}{m^2}$ $\frac{C}{m^2}$

Normal components of electric flux densities are discontinious at the boundary surface by an amount equal to the surface charge density present ~~at~~ on the boundary surface.

$\rho_s = 0$ then $|\vec{D}_{N1}| = |\vec{D}_{N2}|$

$D_{N1} = D_{N2} \Rightarrow$ continious

$$\epsilon_1 \vec{E}_{N1} = \epsilon_2 \vec{E}_{N2}$$

$$\vec{E}_{N1} = \frac{\epsilon_2}{\epsilon_1}\vec{E}_{N2}$$

discontinious

② magnetic boundary conditions

$$\vec{H}_{t1} - \vec{H}_{t2} = \vec{a}_{N12} \times \vec{K}$$

$\left(\frac{A}{m^2}\right)$ $\left(\frac{A}{m^2}\right)$ $\left(\frac{A}{m^2}\right)$

tangential components of magnetic field intensities are discontinious at the boundary surface. by an amount equal to the surface current density present on the boundary surface if $\vec{K} = 0$ then $\vec{H}_{t1} = \vec{H}_{t2} \rightarrow$ continious

$$\frac{\vec{B}_{t1}}{\mu_1} = \frac{\vec{B}_{t2}}{\mu_2}$$

$$\vec{B}_{t1} = \frac{\mu_1}{\mu_2}\vec{B}_{t2} \rightarrow \text{discontinious.}$$

$$\rightarrow \boxed{\vec{B}_{N1} = \vec{B}_{N2}}$$

the Normal compo. of m.f. intensity are continious at boundary surface.

$$\mu_1 \vec{H}_{N1} = \mu_2 \vec{H}_{N2} \Rightarrow \vec{H}_{N1} = \frac{\mu_2}{\mu_1}\vec{H}_{N2} \rightarrow \text{discontinious.}$$

Observation

$z<0$ (medium m_1) | $z>0$

m_2 conductor $\sigma_2 \rightarrow \infty$

$$\vec{J}_2 = \sigma_2 \vec{E}_2$$

$$\vec{E}_2 = \frac{J_2}{\sigma_2 \rightarrow \infty} = 0$$

$$\vec{E}_2 = 0$$

$$\vec{E}_{t2} + \vec{E}_{N2} = 0$$

$$\vec{E}_{t2} = 0 \rightarrow (1)$$

$$\vec{E}_{N2} = 0 \rightarrow (2)$$

Note* (1) b/w Dielectric (m_1) and conductor (m_2) boundary

m_1 dielectric	m_2 Conductor
$\boxed{\vec{E}_{t1} = 0}$	$\boxed{E_{t2} = 0}$ $\boxed{E_{N2} = 0}$
$\boxed{\lvert\vec{D}_{N1}\rvert = \rho_s}$	

boundary

Q $\mu_1 = 2\,\mu H/m$ in region m1 ($z>0$)

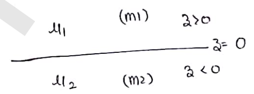
$\mu_2 = 7\,\mu H/m$ in region 2 ($z<0$)

$\vec{J}$ $\vec{K} = 40\hat{a}_x$ A/m on $z=0$ surface

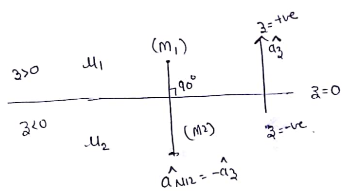
$\vec{B}_1 = 3\hat{a}_x - 2\hat{a}_y - 6\hat{a}_z$ Tesla

Find $\vec{B}_2$

$\vec{J}$ is same $\vec{K}$ see the unit dont confuse by J by current density



Sol^n



$\rightarrow$ For $z=0$ boundary $\hat{a}_z$ is Normal component

$$\vec{B}_{N1} = -6\hat{a}_z \qquad \vec{B}_{t1} = 3\hat{a}_x - 2\hat{a}_y$$

$$\rightarrow \vec{H}_{t1} - \vec{H}_{t2} = \vec{a}_{N12} \times \vec{K}$$

$$\frac{\vec{B}_{t1}}{\mu_1} - \frac{\vec{B}_{t2}}{\mu_2} = \vec{a}_{N12} \times \vec{K}$$

$$\vec{B}_{t2} = \mu_1\left(\frac{B_{t1}}{\mu_1} - \vec{a}_{N12} \times \vec{K}\right)$$

$$\vec{B}_{t2} = 2\times10^{-6}\left[\left(\frac{3\hat{a}_x - 2\hat{a}_y}{7\times10^{-6}}\right) - (-\hat{a}_z)\times 40\hat{a}_x\right] \rightarrow (1)$$

$\vec{B}_{N1} = \vec{B}_{N2}$

$\vec{B}_{N2} = -6\hat{a}_z$ — 2

→ $\vec{B}_2 = \vec{B}_{t2} + \vec{B}_{N2}$ — 3

1, 2 in 3

Maxwell equations

objective qu^n:-
maxwell modified amp. circuital law on the basis of continuity eq^n

1 ampere's circuital law

$$\oint \vec{H} \cdot \vec{dl} = I \text{ — } 1$$

stokes theorem

I dyn

$$\iint (\nabla \times \vec{H}) \cdot \vec{dS} = \iint \vec{J} \cdot \vec{dS}$$

Compare ⇒ $\vec{\nabla} \times \vec{H} = \vec{J} \rightarrow 2$

eq^n 2 is applicable for static field

→ for time varying fields add some term to eq^n 2

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{X} \text{ — } 3$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \nabla \cdot \vec{J} + \vec{\nabla} \cdot \vec{X}$$

$$0 = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{X}$$

$$\vec{\nabla} \cdot \vec{X} = -\vec{\nabla} \cdot \vec{J} \rightarrow 4$$

continuity eq^n is

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \rightarrow 5$$

5 in 4 ⇒ $\vec{\nabla} \cdot \vec{X} = \frac{\partial \rho_v}{\partial t} \rightarrow 6$

$\rho_v = \vec{\nabla} \cdot \vec{D}$ — 7

7 in 6

$$\vec{\nabla} \cdot \vec{X} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

compare $\vec{X} = \frac{\partial \vec{D}}{\partial t} \rightarrow 8$

8 in 3

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$J_c + J_d = \vec{J_t}$$

$\boxed{\text{objective Qn}}$

$$\vec{J_t} = \vec{J_c} + \vec{J_d}$$

$$J_t = \sigma\vec{E} + \epsilon\frac{\partial \vec{E}}{\partial t}$$

for cond^c: 10^8, 10^{-12}

for cond^r: $\boxed{J_t \simeq \sigma \vec{E}}$

for dielectric: $\sigma = 10^{-17}$, 10^{-12}

$$\boxed{J_t \simeq \epsilon \frac{\partial E}{\partial t}}$$

J_c = Conduction current density.

J_d = displacement current density.

J_t = total current density.

Maxwell eq^n

Differential form | Integral form

① $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ | $\oint \vec{E} \cdot \vec{dl} = \iint \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot \vec{ds}$ (Faraday law)

② $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ | $\oint \vec{H} \cdot \vec{dl} = \iint \vec{J} \cdot \vec{ds} + \iint \left(\frac{\partial \vec{D}}{\partial t}\right) \cdot \vec{ds}$ (Amperes circuital Law)

$= I_c + I_d$

③ $\vec{\nabla} \cdot \vec{D} = \rho_v$ | $\oiint \vec{D} \cdot \vec{ds} = \phi$ [Gauss Law of electric field]

④ $\vec{\nabla} \cdot \vec{B} = 0$ | $\oiint \vec{B} \cdot \vec{ds} = 0$ [" " of magnetic "]

① Linear medium

$\vec{J} = \sigma \vec{E}$; $\vec{B} = \mu \vec{H}$; $\vec{D} = \epsilon \vec{E}$

(ii) Homogeneous medium

ϵ, μ, σ are constants

→ non Homogeneous medium

ϵ, μ, σ are func of x, y, z

(iii) Isotropic medium ϵ, μ, σ

values dont change with dir^n

→ non isotropic (or) anisotropic medium

$\epsilon\ \mu\ \sigma$ values change with medium

(iv) charge free medium $\rho_v = 0$

Note* The following maxwell eq^n describe the radiation of EM waves

$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ $\boxed{\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}}$

E(t) produce H(t)
H(t) produce E(t)